

GEORGE PÓLYA AND THE HEURISTIC TRADITION

Tibor FRANK

Eötvös Loránd University – Hungria

[Currently: Max-Planck-Institut für Wissenschaftsgeschichte, Berlin, Germany]

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Resumo

George Pólya (1887-1985) foi um dos maiores educadores matemáticos de todos os tempos. Nascido na Hungria, ele trabalhou como professor primeiramente na Suíça e resolveu imigrar para os Estados Unidos pouco antes das ameaças sofridas pela Segunda Guerra Mundial. Seu método de resolução de problemas não foi pioneiro em matemática, mas contribuiu para a teoria de conhecimento. Vários dos livros seus foram traduzidos para diferentes idiomas, particularmente *How to Solve It* (1945), um clássico nesta área. Estruturado com base em alguns membros famosos da comunidade húngara, e contemporâneos de Pólya, como Theodore von Kármán, John von Neumann, Michael Polanyi, Leo Szilard, Edward Teller, e Nobel Eugene Wigner Laureado, o artigo mostra como o pensamento heurístico tornou-se uma ferramenta e um método para uma geração inteira de cientistas. Traçando as origens deste método à matemática e à educação científica em Budapeste, por volta do século XIX, o autor mostra como até mesmo a Competição Matemática de Stanford (1946-1965) estava arraigada na Competição de Eötvös, organizada em Budapeste desde 1894. Críticos tinham razão ao notar que a influência de Pólya se estendia além da comunidade da educação matemática: em muitos casos ele ajudou na transferência da grande tradição liberal de raciocínio independente e da tática de resolver problemas da Hungria e do Império Austro-húngaro para os Estados Unidos.

Palavras-chave: George Pólya; Resolução de Problemas; Heurística.

Abstract

George Pólya (1887-1985) was one of the greatest mathematics educators of all times. Born in Hungary, he taught first in Switzerland and settled over to the United States just before World War II threatened him in Europe. His method of problem solving was not just pioneering in mathematics but contributed to the theory of knowledge. Several of his books have been translated into dozens of languages, particularly *How to Solve It* (1945), a classic in its own field. Setting Pólya against the background of his famous fellow-Hungarian contemporaries such as Theodore von Kármán, John von Neumann, Michael Polanyi, Leo Szilard, Edward Teller, and Nobel Laureate Eugene Wigner, the article shows how heuristic thinking became a shared tool and method of an entire generation of scientists. Tracing the

origins of this method to the excellent mathematics and science education in Budapest at the turn of the 19th century, the author shows how even the Stanford Mathematics Competition (1946-1965) was rooted in the Eötvös Competition organized in Budapest from 1894 onwards. Critics were right to note that Pólya's influence extended "far beyond the mathematics education community:" in many ways he helped transfer the great liberal tradition of independent reasoning and the tactic of problem solving from Hungary and the Austro-Hungarian Monarchy to the United States.

Keywords: George Pólya; Problem Solving; Heuristics.

Fascination with Genius in Central Europe

Mental processes, the concept and structure of cognition received increasingly special attention in *fin-de-siècle* Central Europe. The turn of the century was fascinated and, indeed, intrigued by genius, and the subject of scientific discovery and problem solving seemed particularly relevant in Germany and the Austro-Hungarian Monarchy, well before World War I. Italian psychiatrist Cesare Lombroso's landmark study on genius and insanity (*Genio e follia*, 1864) was translated into German in 1887, his *L'uomo di genio in rapporto alla psichiatria* (1889) in 1890. Hermann Türck published a highly successful study on genius in 1896 in Berlin, Albert Reibmayer described talent and genius in Munich in 1908 in two volumes, and Wilhelm Ostwald studied the biology of genius in Leipzig in 1910. Ernst Kretschmer published his 1919 Marburg university lectures on genius in 1929, shortly after the appearance of Wilhelm Lange-Eichbaum's volume on genius and madness.¹ Research in Germany obviously influenced, or at least coincided with, Lewis M. Terman's Stanford studies on genius. Actually, both the German and the American studies on intelligence were based largely on the French Binet-Simon intelligence test, which was adapted for the needs of several countries (for example, the Stanford-Binet Scale developed by Terman in the United States, as well as the tests by Otto Bobertag in Germany, Gustav Jaederholm in Sweden, and Mátyás Éltés in Hungary). Considerable interest was shown in the subject in contemporary Hungary, as indicated by Henriette von Szirmay-Pulszky's study of genius and insanity among Hungarian intellectuals² as well as József Somogyi's

¹ Cesare Lombroso, *Genie und Irrsin* (Übersetzt von A. Courth; Leipzig: Reclam, 1887); Hermann Türck, *Der geniale Mensch* (7. Aufl., Berlin: Dümmlers, 1910); Dr. Albert Reibmayer, *Die Entwicklungsgeschichte des Talentes und Genies*, Vols. I-II (München: J. F. Lehmanns, 1908); Wilhelm Ostwald, *Grosse Männer* (Leipzig: Akademische Verlagsgesellschaft m.b.H., 1910); W. Lange-Eichbaum, *Genie, Irrsinn und Ruhm* (München: Ernst Reinhardt, 1928); Ernst Kretschmer, *Geniale Menschen* (2. Aufl. Berlin: Julius Springer, 1931); W. Lange-Eichbaum, *Genie, Irrsinn und Ruhm* (München: Ernst Reinhardt, 1928; new ed., with Wolfram Kurth, München-Basel: Ernst Reinhardt Verlag, 1967/1979, repr. Frechen: Komet, n. d.); W. Lange-Eichbaum, *Das Genie-Problem* (München: Ernst Reinhardt, 1931).

² H. von Szirmay-Pulszky, *Genie und Irrsinn im Ungarischen Geistesleben* (München: Ernst Reinhardt, 1935).

book on talent and eugenics.³ Psychologist Géza Révész studied talent and genius throughout his career, culminating in his 1952 book *Talent und Genie*.⁴

To be sure, Central Europe was perplexed by the secrets of the mind and its workings, and the processes of understanding/knowing, intuition/perception, intelligence/intellect came to be recognized as central issues in the sciences and humanities of German-speaking Europe. In 1935 Karl Duncker of the University of Berlin provided a summary of the psychology of productive thinking.⁵ To those trained by the German literature on the subject, including several generations of Hungarian scientists and scholars, the plethora of work done on productive thinking in German provided copious introductions to the theory of knowledge, the biology of talent, and the philosophy of problem solving. Much of the interest in the theory of knowledge and of knowing was generated in Vienna, where philosophers such as Professors Ernst Mach and Ludwig Boltzmann contributed significantly to the development of a scientific interpretation of the workings of the mind. Mach's main concern was the relationship between everyday thinking and scientific reasoning.⁶ Franz Brentano and his students Kasimir Twardowski and Christian von Ehrenfels were active in the field of phenomenology and knowledge and played an important role in the philosophical study of the language.⁷ From Vienna these new ideas and trends quickly spread to Budapest.

Mach's work had considerable influence on contemporary European philosophers and scientists such as the English Sir Oliver Lodge and Karl Pearson, the Russian Alexander Bogdanov, and the Austrian Friedrich Adler, the assassin of Austrian prime minister Count Karl von Stürgkh. These works became a target of vicious critical attack by Vladimir I. Lenin in his defense of Marxism in 1908 for "the old absurdity of philosophical subjective idealism."⁸ It is remarkable how the anti-Marxist, non-Marxist, pseudo-Marxist scholarship, and particularly Ernst Mach's work, influenced the philosophical tradition in

³ Dr. József Somogyi, *Tehetség és eugenika. A tehetség biológiai, pszichológiai és szociológiai vizsgálata* [Talent and Eugenics. The Biological, Psychological, and Sociological Study of Talent] (Budapest: Eggenberger, 1934).

⁴ Géza Révész, *Das frühzeitige Auftreten der Begabung und ihre Erkennung* (Leipzig: J. A. Barth, 1921), *The Psychology of a Musical Prodigy* (London: Routledge, 1925, repr. 1999), *Das Schöpferisch-persönliche und das Kollektive in ihrem kulturhistorischen Zusammenhang* (Tübingen: Mohr, 1933), *Talent und Genie: Grundzüge einer Begabungspsychologie* (Bern: Francke, 1952).

⁵ Dr. Karl Duncker, *Zur Psychologie des produktiven Denkens* (Berlin: Julius Springer, 1935).

⁶ Ernst Mach, *Erkenntnis und Irrtum. Skizzen zur Psychologie der Forschung* (2nd ed. Leipzig: Barth, 1906), p. XI.

⁷ Peter Weibel, "Das Goldene Quadrupel: Physik, Philosophie, Erkenntnistheorie, Sprachkritik. Die Schwelle des 20. Jahrhunderts: Wissenschaftliche Weltauffassung in Wien um 1900," in: *Wien um 1900. Kunst und Kultur* (Wien-München: Christian Brandstätter, 1985), 407-418; J. C. Nyíri, "Ehrenfels und Masaryk: Überlegungen an der Peripherie der Geschichte." In: *Am Rande Europas. Studien zur österreichisch-ungarischen Philosophiegeschichte* (Wien: Böhlau Verlag, 1988), pp. 40-67.

⁸ V. I. Lenin, *Materialism and Empirio-Criticism. Critical Comments on a Reactionary Philosophy* (1st ed. 1908; London: Lawrence and Wishart, 1950), p. 93.

Central Europe, including Germany, Austria, and Hungary.⁹ Apart from the actual content of Mach's studies, their philosophical and political implications were also relevant in the region, making a lasting impact on liberal thinkers who endeavored to maintain an anti-totalitarian stance in an age of political and doctrinal dictatorships. Albert Einstein extensively used Mach's epistemology and physics, including "Mach's Principle," in his theory of general relativity.¹⁰

The anti-Marxian roots of liberal thought contributed to the estrangement of Hungarian émigré scholars and scientists such as Michael Polanyi and Oscar Jászi after the Soviet takeover of 1945 and contributed in turn to their anti-Soviet attitudes. Apart from directly political reasons, this framework may be helpful in understanding the seemingly unconditional support given to the U. S. military and to NATO during the cold war period by scientists such as John von Neumann, Theodore von Kármán, Karl Mannheim, and, most notably, Edward Teller. The philosophical underpinnings of the anti-totalitarian politics of Hungary's émigré professionals can thus be traced to the traditional idealistic approach to science in Central Europe and the corresponding *Weltanschauung*, a legacy emanating from the philosopher George Berkeley through Albert Einstein.

The notion of a new type of learning, utilizing problem solving and the heuristic method came to be proposed by European immigrant scientists and mathematicians, several of them Hungarians. By the end of World War I, young Karl Mannheim had already written his doctoral dissertation in Budapest on the structural analysis of the theory of knowledge. The dissertation became well known after being published in German in 1922 as *Die Strukturanalyse der Erkenntnistheorie*. Mannheim drew heavily on the work of the Hungarian philosopher Béla Zalai, who, though largely forgotten today, was instrumental in presenting the question of systematization as a central issue in Hungarian philosophy. In 1918 Mannheim referred to a 1911 article by Zalai on the problem of philosophical systematization.¹¹

⁹ Péter Hanák, "Ernst Mach und die Position des Phänomenalismus in der Wissenschaftsgeschichte," in Fritz Klein, Hg., *Europa um 1900. Texte eines Kolloquiums* [Association Internationale d'Histoire Contemporaine de l'Europe] (Berlin: Akademie Verlag, 1989), pp. 265-282.

¹⁰ *The Encyclopaedia Britannica* (Chicago: Encyclopaedia Britannica, 1990), Vol. 7, p. 631; cf. Albert Einstein, "Principles of Research," Address before the Physical Society in Berlin, 1918; "Geometry and Experience," Lecture before the Prussian Academy of Sciences, January 27, 1921; "On the Theory of Relativity," Lecture at King's College in London, 1921; "Physics and Reality," *The Journal of the Franklin Institute*, Vol. 221, No. 3, March 1936, republished in Albert Einstein, *Ideas and Opinions* (New York: Bonanza Books, 1954), pp. 227, 239, 248, 303.

¹¹ Karl Mannheim, *Die Strukturanalyse der Erkenntnistheorie*, Kant-Studien, Ergänzungshefte, No. 57, Berlin: Reuther & Reichard, 1922. (Hungarian original: *Az ismeretelmélet szerkezeti elemzése*, Budapest: Athenaeum, 1918); Béla Zalai, "A filozófiai rendszerezés problémái," [The Problem of Philosophical Systematization], *A Szellem*, 1911, No. 2, pp. 159-186; Vilmos Szilasi, *A tudati rendszerezés elméletéről. Bevezetés* [On the Theory of Systematization of the Mind. An Introduction] A Magyar Filozófiai Társaság Könyvtára, Vol. II (Budapest: Franklin, 1919) Cf. Otto Beöthy, "Zalai Béla (1882-1915). Egy pálya emlékezete," [Béla Zalai (1882-1915). The Memory of a Life], in: Endre Kiss and Kristóf János Nyíri, eds., *A magyar filozófiai gondolkodás a századelőn* [Hungarian Philosophy at the Turn of the Century] (Budapest: Kossuth, 1977), pp. 228-231.

In a related field, heuristics was described as a “tactics of problem solving,” “an interdisciplinary no man’s land which could be claimed by scientists and philosophers, logicians and psychologists, educationalists and computer experts.”¹² Fascination with the subject among émigré Hungarians is probably best demonstrated by three important books by the author Arthur Koestler. Sharing the background of many of the Hungarian scientists in exile, Koestler was intrigued by the “act of creation” for a long time after World War II (*Insight and Outlook*, 1949; *The Sleepwalkers*, 1959; *The Act of Creation*, 1964). While working on these books, Koestler regularly consulted some of his illustrious Hungarian friends in England, such as Nobel laureate Dennis Gabor or Michael Polanyi and Koestler once went to Stanford specifically to discuss the matter with Hungarian-American mathematician George Pólya.¹³ The tradition of heuristics is deeply European, with roots in antiquity (Euclid, Pappus, and Proclus) and with forerunners such as Descartes and Leibniz. Heuristic thinking reached the Habsburg empire relatively early in the nineteenth century when it became part of Bernard Bolzano’s philosophy: his *Wissenschaftslehre* (1837) already contained an extensive chapter on *Erfindungskunst*, meaning heuristics. Through the questionable services of his disciple Robert Zimmermann, who possibly plagiarized much of Bolzano’s original book and published many of his master’s ideas under his own name in a popular and widespread textbook called *Philosophische Propädeutik* (1853), these ideas reached a wide audience, and *Erfindungskunst* became an integral part of the philosophical canon of the Habsburg monarchy just before the great generation of scientists and scholars was about to be born.¹⁴

The American Origins of Problem Solving

The origins of problem solving as a way of thinking particularly associated with the American heritage go back to the pioneering spirit of the frontier times. Its value was first recognized as a necessity for the development of tactics and strategies that the

¹² George Polya, “Methodology or Heuristics, Strategy or Tactics?” *Archives de Philosophie*, Tome 34, Cahier 4, Octobre-Décembre 1971, pp. 623-629, quote p. 624.

¹³ Arthur Koestler, *Insight and Outlook: An Inquiry into the Common Foundations of Science, Art and Social Ethics* (New York: Macmillan, 1949); *The Sleepwalkers: A History of Man’s Changing Vision of the Universe* (New York: Grosset & Dunlap, 1959); *The Act of Creation* (New York: Macmillan, 1964). Cf. M[enachen] M. Schiffer, “George Polya, 1887-1985,” George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA; cf. Arthur Koestler, *The Act of Creation*, *op. cit.* p. 23; Béla Hidegkuti, “Arthur Koestler and Michael Polanyi: Two Hungarian Minds in Partnership in Britain,” *Polanyiana*, Vol. 4, No. 4, Winter 1995, pp. 1-81.

¹⁴ Eduard Winter, Hg., *Robert Zimmermanns Philosophische Propädeutik und die Vorlagen aus der Wissenschaftslehre Bernard Bolzanos*. Eine Dokumentation zur Geschichte des Denkens und der Erziehung in der Donaumonarchie (Wien: Böhlau Verlag, 1975), pp. 7-36. Cf. Bernard Bolzanos *Wissenschaftslehre*: Versuch einer ausführlichen und grösstentheils neuen Darstellung der Logik mit steter Rücksicht auf deren bisherige Bearbeiter (Sulzbach: J. E. v. Seidel, 1837).

American people could use to perform their daily tasks in creating the United States, and their own lives in the new country.

The first serious American psychologist to emphasize learning as the definition of intelligence was Edward L. Thorndike (1874-1949), who in a 1921 symposium defined intelligence as the ability to give good responses to questions.¹⁵ Thorndike was influenced by William James at Harvard (1895-1897). The concept of learning took on practical characteristics in regard to problem solving as pertaining not only to survival in the harsh circumstances of the frontier, but also to the newly emerging American tradition of self-help and success. This became an ideal upheld by subsequent generations of American social scientists, authors, businessmen throughout the latter half of the 19th century. The doctrine of self-help and success became, indeed, an American myth and this dream "penetrated, in some guise, every major activity of the period: the new immigration, the rise of socialism, the agrarian revolution (and the exodus of country boys and girls to the city), the march of technology, the growth of corporate business and labor unionism, and more."¹⁶

In a pioneering inquiry into the nature of problems and the solution of a problem, Michael Polanyi defined one of the most crucial questions of his generation. European mathematicians and scientists were particularly welcome in the United States from the 1920s onwards. "To recognize a problem which can be solved and is worth solving is in fact a discovery in its own right," Polanyi declared the creed of his generation in his 1957 article for *The British Journal for the Philosophy of Science*.¹⁷ Polanyi spoke for, and spoke of, his generation when discussing originality and invention, discovery and heuristic act, investigation and problem solving. The interpretative frame of the educated mind is ever ready to meet somewhat novel experiences and to deal with them in a somewhat novel manner. In this sense, all life is endowed with originality and originality of a higher order is but a magnified form of a universal biological adaptivity. But genius makes contact with reality on an exceptionally wide range: by seeing problems and reaching out to hidden possibilities for solving them, far beyond the anticipatory powers of current conceptions. Moreover, by deploying such powers in an exceptional measure - far surpassing our own as onlookers - the work of genius offers us a massive demonstration of a creativity which can never be explained in other terms nor taken unquestioningly for granted.¹⁸

¹⁵ R[obert] J. St[ernberg], "Human Intelligence", *The New Encyclopaedia Britannica* (Chicago: Encyclopaedia Britannica, 1990), Vol. 21, p. 710. Cf. Donald O. Hebb, *Textbook of Psychology* (Philadelphia—London—Toronto: W. B. Saunders, 1972).

¹⁶ Kermit Vanderbilt, "The Gospel of Self-Help and Success in the Gilded Age", in: Robert Allen Skotheim, Michael McGiffert, eds. *American Social Thought: Sources and Interpretations*, Vol. II: *Since the Civil War* (Reading, MA: Addison-Wesley, 1972), p. 6.

¹⁷ Michael Polanyi, "Problem Solving", *The British Journal for the Philosophy of Science*, Vol. VIII, No. 30, August 1957, pp. 89-103; quote p. 89.

¹⁸ Polanyi, "Problem Solving", *op. cit.*, pp. 93-94.

„How to Solve It“

Hungarian-born mathematician George Pólya was one of those who channeled the Hungarian and, more broadly speaking, European school tradition into American education in a series of books and articles, starting with his 1945 book *How to Solve It*.¹⁹ In 1944 Pólya remembered the time when, at the turn of the century in Hungary,

he was a student himself, a somewhat ambitious student, eager to understand a little mathematics and physics. He listened to lectures, read books, tried to take in the solutions and facts presented, but there was a question that disturbed him again and again: ‘Yes, the solution seems to work, it appears to be correct; but how is it possible to invent such a solution? Yes, this experiment seems to work, this appears to be a fact; but how can people discover such facts? And how could I invent or discover such things by myself?’²⁰

Pólya came from a distinguished family of academics and professionals. His father, Jakab, an eminent lawyer and economist provided the best education for his children. They included George’s brother, Jenő Pólya, the internationally recognized Hungarian professor of surgery and honorary member of the American College of Surgeons.²¹ George Pólya first studied law, later changing to languages and literature, then philosophy and physics, to settle finally on mathematics, in which he received his Ph.D. in 1912. He was a student of Lipót Fejér, whom Pólya considered one of the key people who influenced Hungarian mathematics in a definitive way.

Pólya felt that Fejér, the competitive examination in mathematics and the Hungarian mathematical journal *Középiskolai Matematikai Lapok* (High School Papers on Mathematics) were responsible for the development of a large number of major mathematicians in Hungary.²²

For emancipated Jews in Hungary, who received full rights as citizens in 1867, it was ultimately the Hungarian Law 1867:XII that made it possible, among other things, to become teachers in high schools and even professors at universities. This is one of the reasons that lead to the explosion of mathematical talent in Hungary, just as happened in

¹⁹ G. Pólya, *How to Solve it. A New Aspect of Mathematical Method* (Princeton, N.J.: Princeton University Press, 1945). *How to Solve It* has never been out of print and has sold well over 1 million copies. It has been translated into 17 languages, probably a record for a modern mathematics book. Gerald L. Alexanderson, “Obituary. George Pólya,” *Bulletin of the London Mathematical Society*, Vol. 19, 1987, p. 563, 603.

²⁰ G. Pólya, “*How to Solve It*,” op. cit., p. vi.

²¹ Vilmos Milkó, Pólya Jenő emlékezete [In memoriam Jenő Pólya], *Archivum Chirurgicum*, Vol. 1, No. 1, 1948, p.1.

²² G. Pólya, “Leopold Fejér,” *Journal of the London Mathematical Society*, Vol. 36., 1961, p. 501.

Prussia after the emancipation of Jews in 1812.²³ John Horváth of the University of Maryland was one who pointed out the overwhelming majority of Jewish mathematicians in Hungary in the early part of the 20th century.

Culture in the second half of the nineteenth century became a matter of very high prestige in Hungary, where the tradition to respect scientific work started to loom large after the Austro-Hungarian *Ausgleich* (Compromise or Settlement) in 1867 between Austria and Hungary. For sons of aspiring Jewish families, a professorship at a Budapest university or membership in the Hungarian Academy of Letters and Science promised entry into the Hungarian élite and eventual social acceptance in Hungarian high society, an acknowledged way to respectability. Pursuing scientific professions, particularly mathematics, secured a much desired social position for sons of Jewish-Hungarian families, who longed not only for emancipation, but for full equality in terms of social status and psychological comfort. Thus, in many middle class Jewish families, at least one of the sons was directed into pursuing a career in academe.

Distinguished scientists such as Manó Beke, Lipót Fejér, Mihály Fekete, Alfréd Haar, Gyula and Dénes König, Gusztáv Rados, Mór Réthy, Frigyes Riesz, and Lajos Schlesinger belonged to a remarkable group of Jewish-Hungarian mathematical talents, who, after studying at major German universities, typically Göttingen or Heidelberg, became professors in Hungary's growing number of universities before World War I. A few of them, like Gyula König and Gusztáv Rados, even became university presidents at the Technical University of Budapest. There were several other renowned scientists active in related fields, such as physicist Ferenc Wittmann, engineer Donát Bánki and some others. Mathematicians were also needed outside the academic world: just before the outbreak of World War I George Pólya was about to join one of Hungary's big banks, at the age of 26, with a Ph.D. in mathematics and a working knowledge of four foreign languages in which he already published important articles.²⁴

Despite what we know about the social conditions which nurtured and even forced out the talent of these many extraordinary scientists, how this occurred still remains somewhat mysterious. Stanislaw Ulam recorded an interesting conversation with John

²³ R. Hersch and V. John-Steiner, "A Visit to Hungarian Mathematics," Ms., pp. 35-37.

I received a copy of this article from Professor Gerald L. Alexanderson of the Department of Mathematics, Santa Clara University, Santa Clara, CA. John Horváth compared this explosion of Jewish talent after the Jewish emancipation to the surprising number of sons of Protestant ministers entering the mathematical profession in Hungary after World War II, "Those kids would have become Protestant ministers, just as the old ones would have become rabbis... mathematics is the kind of occupation where you sit at your desk and read. Instead of reading the Talmud, you read proofs and conjectures. It's really a very similar occupation." R. Hersch and V. John-Steiner, *op. cit.*, p.37.

²⁴ György Pólya to Baron Gyula Madarassy-Beck, Paris, February 23, 1914. I am grateful to Professor Gerald Alexanderson of the University of Santa Clara for showing me this document as well as his collection of Pólya documents that were to be transferred to the George Pólya Papers, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

von Neumann when describing their 1938 journey to Hungary in his *Adventures of a Mathematician*.

I returned to Poland by train from Lillafüred, traveling through the Carpathian foothills. . . This whole region on both sides of the Carpathian Mountains, which was part of Hungary, Czechoslovakia, and Poland, was the home of many Jews. Johnny [von Neumann] used to say that all the famous Jewish scientists, artists, and writers who emigrated from Hungary around the time of the first World War came, either directly or indirectly, from these little Carpathian communities, moving up to Budapest as their material conditions improved. The [Nobel Laureate] physicist I[sidor] I[saac] Rabi²⁵ was born in that region and brought to America as an infant. Johnny used to say that it was a coincidence of some cultural factors which he could not make precise: an external pressure on the whole society of this part of Central Europe, a feeling of extreme insecurity in the individuals, and the necessity to produce the unusual or else face extinction.²⁶

An interesting fact about some of the Jewish-Hungarian geniuses at the turn of the century was that some of them could multiply huge numbers in their head. This was true of von Kármán, von Neumann and Edward Teller. Von Neumann, in particular, commanded extraordinary mathematical abilities. Nevertheless, there is no means available to prove that this prodigious biological potential was more present in Hungary at the turn of the century than elsewhere in Europe.²⁷

Similarly, heuristic thinking was also a common tradition that many other Hungarian mathematicians and scientists shared. John Von Neumann's brother remembered the mathematician's "heuristic insights" as a specific feature that evolved during his Hungarian childhood and appeared explicitly in the work of the mature scientist.²⁸ Von Neumann's famous high school director, physics professor Sándor Mikola, made a special

²⁵ Nobel Prize in Physics, 1944.

²⁶ S. M. Ulam, *Adventures of a Mathematician* (New York: Scribner's, 1976), p. 111. Cf. Tibor Fabian, "Carpathians Were a Cradle of Scientists," Princeton, NJ, November 16, 1989, *The New York Times*, December 2, 1989. -- George Pólya's nephew John Béla Pólya had an even more surprising, though cautious proposition to make. He suggested that through George Pólya's mother, Anna Deutsch (1853-1939), Pólya was related to Eugene Wigner and Edward Teller, "who are thought to have" ancestry originating from the same region between the towns of Arad and Lugos in Transylvania (then Hungary, today Romania). Though this relationship is not yet documented and should be taken at this point merely as a piece of Pólya family legend, it is nonetheless an interesting reflexion of the strong belief in the productivity of the Jewish community in North-Eastern Hungary and Transylvania in terms of mathematical talent. John Béla Pólya, "Notes on George Pólya's family," attached to John Béla Pólya to Gerald L. Alexanderson, Greensborough, Australia, July 28, 1986. -- I am deeply grateful to Gerald L. Alexanderson of Santa Clara University, Santa Clara, CA, for his generous and highly informative support of my research on George Pólya in 1988 and after.

²⁷ Norman Macrae, *John von Neumann* (New York: Pantheon, 1992), p. 9; J. M. Rosenberg, *Computer Prophets* (New York: Macmillan, 1969) p. 155. ff.; Edward Teller and Alan Brown, *The Legacy of Hiroshima* (Garden City: Doubleday, 1962) p. 160. Cf. William O. McCagg, *op. cit.*, 211.

²⁸ Nicholas A. Vonneuman, *John Von Neumann as Seen by His Brother* (Meadowbrook, PA, 1987), p. 44.

effort to introduce heuristic thinking in the elementary school curriculum in Hungary already in the 1900s.²⁹

Fejér drew a number of gifted students to his circle, such as Mihály Fekete, Ottó Szász, Gábor Szegő and, later, Paul Erdős. His students remembered Fejér's lectures and seminars as "the center of their formative circle, its ideal and focal point, its very soul." "There was hardly an intelligent, let alone a gifted, student who could exempt himself from the magic of his lectures. They could not resist imitating his stress patterns and gestures, such as his personal impact upon them."³⁰ George Pólya remembered Fejér's personal charm and personal drive to have been responsible for his great impact: "F[ejér] influenced more than any other single person the development of math[ematic]'s in Hungary. . ."³¹

In Budapest, Pólya was one of the founders, along with Károly Polányi, of the student society called Galilei Kör [Galileo Circle], where he lectured on Ernst Mach. The Galileo Circle (1908-1918) was the meeting place of radical intellectuals, mostly Jewish college students from the up and coming Budapest families of a new bourgeoisie. Members of the circle became increasingly radical and politicized. Oddly enough, the Communists of 1919 found it far too liberal, while the extremist right-wing régime of Admiral Horthy after 1919 considered it simply Jewish. In a Hungary of varied totalitarian systems, the radical-liberal tradition remained unwanted.³² Soon, however, Pólya went to Vienna where he served the academic year of 1911, after receiving his doctorate in mathematics in Budapest. In 1912-13 he went to Göttingen, and later to Paris and Zurich, where he took an appointment at the Eidgenössische Technische Hochschule (Swiss Federal Institute of Technology). He became full professor at the ETH in 1928.

A distinguished mathematician, Pólya drew on several decades of teaching mathematics based on new approaches to problem solving, first as a professor in Zurich,

²⁹ Sándor Mikola, "Die heuristischen Methode im Unterricht der Mathematik der unteren Stufe," in E. Beke und S. Mikola, Hg., *Abhandlungen über die Reform des mathematischen Unterrichts in Ungarn* (Leipzig und Berlin: Teubner, 1911), pp. 57-73.

³⁰ Gábor Szegő, "[Lipót Fejér]," MS. Gábor Szegő Papers, SC 323, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

³¹ [Lecture outline, n.d. unpublished MS] George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

³² Zsigmond Kende, *A Galilei Kör megalakulása* [The Foundation of the Galileo Circle] (Budapest: Akadémiai Kiadó, 1974); Márta Tömöry, *Új vizeken járok: A Galilei Kör története* [Walking on New Waters: A History of the Galileo Circle] (Budapest: Gondolat, 1960); György Litván, *Magyar gondolat -- szabad gondolat* [Hungarian Thought -- Free Thought] (Budapest: Magvető, 1978); György Litván, "Jászi Oszkár, A magyar progresszió és a nemzet," [Oscar Jaszi, Hungarian Progressives and the Nation], in Endre Kiss, Kristóf János Nyíri, eds., *A magyar gondolkodás a századelőn* [Hungarian Philosophy at the Turn of the Century] (Budapest: Kossuth, 1977). Litván pointed out that while similar social science organizations, such as *Társadalomtudományi Társaság* or *Huszadik Század* had a fair number of gentile contributors, the Galileo Circle almost exclusively drew upon progressive Jewish students. Cf. Attila Pók, *A magyarországi radikális demokrata ideológia kialakulása. A "Huszadik Század" társadalomszemlélete (1900-1907)* [The Rise of Democratic Radicalism in Hungary: the Social Concept of *Huszadik Század* (1900-1907)] (Budapest: Akadémiai Kiadó, 1990) p. 152-165.

Switzerland, and later in his life at Stanford, California. It was in Zurich that Pólya and fellow Hungarian Gábor Szegő started their long collaboration by signing a contract in 1923 to publish their much acclaimed joint collection of *Aufgaben und Lehrsätze aus der Analysis*.³³ Considered a mathematical masterpiece even today, *Aufgaben und Lehrsätze* took several years to complete, and it continues to impress mathematicians not only with the range and depth of the problems contained in it, but also with its organization: to group the problems not by subject but by solution method was a novelty.³⁴ His primary concern had always been to provide and maintain “an independence of reasoning during problem solving,”³⁵ an educational goal he declared to be of paramount importance when addressing the Swiss Association of Professors of Mathematics in 1931. Several of his articles on the subject preceded this lecture, probably the earliest being published in 1919.³⁶ Pólya had provided a model for problem solving by the time he was in Berne, Switzerland, suggesting “a systematic collection of rules and methodological advices,” which he considered “heuristics modernized.”³⁷

Pólya was active in a number of important fields of mathematics, such as theory probability, complex analysis, combinatorics, analytic number theory, geometry, and mathematical physics. In the United States after 1940, and at Stanford as of 1942, Pólya became the highest authority on the teaching of problem solving in mathematics.

With his arrival at the United States, Pólya started a new career based on his new found interest in teaching and in heuristics.³⁸ He developed several new courses such as his “Mathematical Methods in Science,” which he first offered in the Autumn 1945 Quarter at Stanford, introducing general and mathematical methods, deduction and induction, the relationship between mathematics and science, as well as the “use of physical intuition in the solution of mathematical problems.”³⁹ In his popular and often repeated Mathematics

³³ Georg Pólya--Gábor Szegő, *Aufgaben und Lehrsätze aus der Analysis* (Berlin: Springer, 1925, new editions: 1945, 1954, 1964, 1970-71), Vols. I-II; Translations: English, 1972-76; Bulgarian, 1973; Russian, 1978; Hungarian, 1980-81.

³⁴ Gerald L. Alexanderson, “Obituary. George Pólya,” *op. cit.*, pp. 562.

³⁵ G. Pólya, “Comment chercher la solution d’un problème de mathématiques?” “*L’enseignement mathématique*, 30e année, 1931, Nos. 4-5-6.

³⁶ G. Pólya, “Geometrische Darstellung einer Gedankenkette”, *Schweizerische Pädagogische Zeitschrift*, 1919.

³⁷ G. Pólya, “Comment chercher,” *op. cit.*

³⁸ Gerald L. Alexanderson, “Obituary. George Pólya,” *op. cit.*, p. 563; on “Pólya the mathematician and teacher,” see pp. 566-570.

³⁹ Paul Kirkpatrick, Acting Dean, School of Physical Sciences, Stanford University, Course outline, September 4, 1945, George Pólya Papers, SC 337, 87-137, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

129 course on “How to Solve the Problem?” Pólya taught mathematical invention and mathematical teaching, quoting English poet and satirist Samuel Butler (1612–80):

All the inventions that the world contains
 Were not by reason first found out, nor brains
 But pass for theirs, who had the luck to light
 Upon them by mistake or oversight.⁴⁰

He surveyed all aspects of a problem, general and specific, restating it in every possible way and pursued various courses that might lead to solving it. He studied several ways to prove a hypothesis or modify the plan, always focusing on finding the solution. He compiled a characteristic list of “typical questions for this course,” which indeed contained his most important learnings from a long European schooling.⁴¹

In a course on heuristics he focused on problems and solutions, using methods from classical logic to heuristic logic. Offering the course alternately as Mathematics 110 and Physical Sciences 115, he sought to attract a variety of students, including those in education, psychology and philosophy.⁴² The courses were based on Pólya’s widely used textbook *How to Solve It*.

In due course, Pólya published several other books on problem solving in mathematics such as the two-volume *Mathematics and Plausible Reasoning* (1954), and *Mathematical Discovery*, in 1965. Both became translated into many languages.⁴³ Towards

⁴⁰ G. Pólya, “Elementary Mathematics from Higher Point of View,” Mathematics 129, George Pólya Papers, SC 337, 86-036, Box 1, Folder 9, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA. Cf. Samuel Butler, “Miscellaneous Thoughts,” in: *The Poems of Samuel Butler*, Vol. II (Chiswick: C. Willingham, 1822), p. 281.

⁴¹ G. Pólya, “Elementary Mathematics from a Higher Point of View,” Survey of Typical Questions, George Pólya Papers, SC 337, 87-137, Box 3, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA. -- Pólya was indeed very well read and liked to show his erudition by quoting Socrates, Descartes, Leibniz, Kant, Herbert Spencer, Thomas Arnold, J. W. von Goethe, Leonhard Euler and his famous colleagues, such as Albert Einstein, and many others. George Pólya Papers, SC 337, 87-034, Box 1&3, 87-137, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁴² Untitled course description, n. d. George Pólya Papers, SC 337, 86-036, Box 1, Folder 3, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁴³ G. Pólya *Mathematics and Plausible Reasoning* (Princeton, N.J.: Princeton University Press, 1954, 2nd ed. 1968), Vols. 1-2. Translations: Bulgarian, 1970; French, 1957-58; German, 1962-63; Japanese, 1959; Romanian, 1962; Russian, 1957, 1975; Spanish, 1966; Turkish, 1966. G. Pólya, *Mathematical Discovery. On Understanding, Learning, and Teaching Problem Solving* (New York-London-Sidney: John Wiley and Sons, 1965, Vols. 1-2; combined paperback ed. 1981), Translations: Bulgarian, 1968; French, 1967; German, 1966, 1967, 1979, 1983; Hungarian, 1969, 1979, Italian, 1970-71, 1979, 1982; Japanese, 1964; Polish, 1975; Romanian, 1971; Russian 1970, 1976. Cf. Gerald L. Alexanderson, “Obituary. George Pólya,” *op. cit.*, pp. 604-605.

the end of his career his “profound influence of mathematical education” was internationally recognized.⁴⁴

Pólya’s significance in general methodology seems to have been his proposition to interpret heuristics as problem solving, more specifically, the search for those elements in a given problem that may help us find the right solution.⁴⁵ For Pólya, heuristics equaled “*Erfindungskunst*,” a way of inventive or imaginative power, the ability to invent new stratagems of learning, and bordered not only on mathematics and philosophy but also psychology and logic. In this way a centuries-old European tradition was renewed and transplanted into the United States where Pólya had tremendous influence on subsequent generations of teachers of mathematics well into the 1970s. In 1971 the aged mathematician received an honorary degree at the University of Waterloo where he addressed the Convocation, appropriately calling for the use of “heuristic proofs”: “In a class for future mathematicians you can do something more sophisticated: You may present first a heuristic proof, and after that a strict proof, the main idea of which was foreshadowed by the heuristic proof. You may so do something important for your students: You may teach them to do research.”⁴⁶ “Heuristics should be given a new goal,” Pólya argued, “that should in no way belong to the realm of the fantastic and the utopian.”⁴⁷

Problem solving for Pólya was seen as “one third mathematics and two thirds common sense.”⁴⁸ This was a tactic which he emphatically suggested for teachers of mathematics in American high schools. If the teaching of mathematics neglects this tactic, he commented, it misses two important goals: “It fails to give the right attitude to future users of mathematics, and it fails to offer an essential ingredient of general education to future non-users of mathematics.”⁴⁹

⁴⁴ A good example was the Second International Congress on Mathematical Education at the University of Exeter, England. Cf. the invitation sent to Pólya by the Chairman of the Congress, Professor Sir James Lighthill, FRS, June 23, 1971. [Cambridge] George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁴⁵ G. Pólya, “Die Heuristik. Versuch einer vernünftigen Zielsetzung,” *Der Mathematikunterricht*, Heft 1/64 (Stuttgart: Ernst Klett, 1964); cf. ‘L’Heuristique est-elle un sujet d’étude raisonnable?’, *La méthode dans les sciences modernes*, ‘Travail et Méthode’, numéro hors série, pp. 279-285.

⁴⁶ G. Pólya, “Guessing and Proving.” Address delivered at the Convocation of the University of Waterloo, October 29, 1971. George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁴⁷ G. Pólya, “Die Heuristik,” *op. cit.*, p. 5

⁴⁸ George Pólya, Untitled note, n. d., George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁴⁹ G. Pólya, “Formation, Not Only Information,” Address at the Mathematical Association of America, George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

Throughout his career as a teacher he strongly opposed believing in what authorities profess. Teachers and principals, he argued, “should use their own experience and their own judgment.”⁵⁰ His matter-of-fact, experience-based reasoning has been repeatedly described in books and articles. He even made two films on the teaching of mathematics (“Let Us Teach Guessing,” an award winner at the American Film Festival in 1968; “Guessing and Proving,” based on an Open University Lecture, Reading, 1962).⁵¹ The most simple and straightforward summary of his ideas on teaching was presented in the preface of a course that he gave at Stanford and subsequently published in 1967. Pólya’s description is the best introduction to heuristic thinking:

Start from something that is familiar or useful or challenging: From some connection with the world around us, from the prospect of some application, from an intuitive idea.

Don’t be afraid of using colloquial language when it is more suggestive than the conventional, precise terminology. In fact, do not introduce technical terms before the student can see the need for them.

Do not enter too early or too far into the heavy details of a proof. Give first a general idea or just the intuitive germ of the proof.

More generally, realize that the natural way to learn is to learn by stages: First, we want to see an outline of the subject, to perceive some concrete source or some possible use. Then, gradually, as soon as we can see more use and connections and interest, we take more willingly the trouble to fill in the details.⁵²

Pólya had lasting influence on a variety of thinkers in and beyond mathematics. The first curriculum recommendation of the [American] National Council of the Teachers of Mathematics suggested that “problem solving be the focus of school mathematics in the 1980s [in the U. S.]. The 1980 NCTM Yearbook, published as *Problem Solving in School Mathematics*, the Mathematical Association of America’s Compendia of Applied Problems and the new editor of the *American Mathematical Monthly*, P. R. Halmos, all called for more use of problems in teaching in 1980.⁵³ Pólya was part of the “problem solving

⁵⁰ George Pólya to Robert J. Griffin, Stanford, June 12, 1962. George Pólya Papers, SC 337, 87-034, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁵¹ George Pólya to Anthony E. Mellor, Harper and Row, Stanford, March 11, 1974; Stanford University News Service, February 17, 1969. George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁵² George Pólya, “Preface,” MS George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁵³ [Untitled MS, n.d. “The organizers’ choice of George Pólya.”] George Pólya Papers, SC 337, 87-034, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA. - P. R. Halmos, “The Heart of Mathematics,” *American Mathematical Monthly*, Vol. 87, 1980, pp. 519-524.

movement” that cut a wide swath in the 1980s.⁵⁴ Philosopher Imre Lakatos, a fellow-Hungarian who described mathematical heuristics as his main field of interest in 1957, acknowledging his debt to Pólya’s influence, and particularly to *How to Solve It*, which he translated into Hungarian.⁵⁵

Critics, however, like mathematician Alan H. Schoenfeld, pointed out that while Pólya’s influence extended “far beyond the mathematics education community,” “the scientific status of Pólya’s work on problem solving strategies has been more problematic.”⁵⁶ Students and instructors often felt that the heuristics-based approach rarely improved the actual problem-solving performance itself. Researchers in artificial intelligence claimed that they were unable to write problem solving programs using Pólya’s heuristics. “We suspect the strategies he describes epiphenomenal rather than real.”⁵⁷ Recent work in cognitive science, however, has provided methods for making Pólya’s strategies more accessible for problem solving instruction. New studies have provided clear evidence that students can significantly improve their problem-solving performance through heuristics.⁵⁸ “It may be possible to program computer knowledge structures capable of supporting heuristic problem-solving strategies of the type Pólya described.”⁵⁹

The Stanford Mathematics Competition

Initiated jointly by Professors George Pólya and Gábor Szegő, one of the most significant Hungarian contributions to the teaching of mathematics was the introduction of

⁵⁴ Alan H. Schoenfeld, “George Pólya and Mathematic Education,” Gerald L. Alexanderson, Lester H. Lange, “Obituary. George Pólya,” *op. cit.*, p. 595. Cf. Rudolf Groner, Marina Groner, Walter F. Bischof, eds., *Methods of Heuristics* (Hillsdale, N.J., London: Lawrence R. L. Baum, 1983); Stephen I. Brown - Marion E. Walter, *The Art of Problem Posing* (Philadelphia, PA: The Franklin Institute Press, 1983).

⁵⁵ Imre Lakatos to Dr. Maier (Rockefeller Foundation), Cambridge, England, May 5, 1957. George Pólya Papers, SC 337, 87-137, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA. -- In turn, Pólya expressed his admiration for Lakatos’s “Proofs and Refutations,” and recommended him as Professor of Logic at the London School of Economics and Political Science, “with special reference to the Philosophy of Mathematics.” George Pólya to Walter Adams (LSE), Stanford, CA, January 13, 1969, George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁵⁶ Alan H. Schoenfeld, “George Pólya and Mathematic Education,” Gerald L. Alexanderson, Lester H. Lange, “Obituary. George Pólya,” *op. cit.*, p. 595.

⁵⁷ Alan H. Schoenfeld, “George Pólya and Mathematic Education,” Gerald L. Alexanderson, Lester H. Lange, “Obituary. George Pólya,” *op. cit.*, p. 596.

⁵⁸ Alan H. Schoenfeld, *Mathematical Problem Solving* (Academic Press, 1985)

⁵⁹ Alan H. Schoenfeld, “George Pólya and Mathematic Education,” Gerald L. Alexanderson, Lester H. Lange, “Obituary. George Pólya,” *op. cit.*, p. 596.

the Stanford Mathematics Competition for high school students. Modelled after the Eötvös Competition organized in Hungary from 1894 on, the main purpose of the competition was to discover talent, and to revive the competitive spirit of the Eötvös Competition, which Szegő himself won in 1912.⁶⁰ This contest was held annually for over 30 years until it was terminated in 1928. Stress was laid on inherent cognitive ability and insight rather than upon memorization and speed. Those who were able to go beyond the question posed were given additional credit. Those who were cognizant of the preponderance of Hungarian mathematicians were tempted to speculate upon the relationship between the Eötvös Prize and “the mathematical fertility of Hungary.”⁶¹ Winners of the Eötvös Prize have included Lipót Fejér, Theodore von Kármán, Alfréd Haar, George Pólya, Frigyes Riesz, Gábor Szegő, and Tibor Radó.

The Stanford competition was started in 1946 and discontinued in 1965 when the Stanford Department of Mathematics turned more towards graduate training.⁶² When first started, the Stanford Examination was administered to 322 participants in 60 California high schools. The last examination, in 1965, was administered to about 1200 participants in over 150 larger schools in seven states from Nevada to Montana. The Stanford University Competitive Examination in Mathematics emphasized “originality and insight rather than routine competence.” Even a typical question required a high degree of ingenuity and the winning student was asked “to demonstrate research ability.”⁶³

Organizers of the competition thought of mathematics “not necessarily as an end in itself, but as an adjunct necessary to the study of any scientific subject.”⁶⁴ It was suggested that ability in mathematical reasoning correlated with success in higher education in any field. Also, the discovery of singularly gifted students helped identify the originality of mind displayed by grappling with difficult problems: mathematical ability was regarded

⁶⁰ G[eorge] Pólya and J[eremy] Kilpatrick, “The Stanford University Competitive Examination in Mathematics,” *American Mathematical Monthly*, Vol. 80, No.6, June-July, 1963, p. 628.

⁶¹ T. Radó, “Mathematical Life in Hungary,” *American Mathematical Monthly*, Vol. XXXIX, 1932, pp. 85-90; József Kürschák, *Matematikai versenytételek* (Budapest, 1929); József Kürschák, *Hungarian Problem Book: Based on the Eötvös Competitions, 1894-1928* (New York: Random House, 1963); R. Creighton Buck, “A Look at Mathematical Competitions,” *American Mathematical Monthly*, Vol. LXVI, No. 3. March 1959, p. 209.

⁶² Department of Mathematics, Stanford University, “The Stanford University Mathematics Examination,” *American Mathematical Monthly*, Vol. LIII, No. 7, August-September, 1946, pp. 406-409. G[eorge] Pólya and J[eremy] Kilpatrick, *op. cit.*, pp. 627-640; cf. the correspondence between Harley Flanders, George Pólya and Jeremy Kilpatrick, 1970-1972, George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁶³ R. Creighton Buck, “A Look at Mathematical Competitions,” *American Mathematical Monthly*, Vol. LXVI, No. 3, March 1959, pp. 204-205.

⁶⁴ H. M. Bacon, “The Stanford University Competitive Examination in Mathematics,” Report at the Meeting of the Mathematical Association of America, University of Washington, Seattle, August 20, 1956. George Pólya Papers, SC 337, 86-036, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

as an index of general capacity.⁶⁵ Those responsible for the competition were firmly convinced that “an early manifestation of mathematical ability is a definite indication of exceptional intelligence and suitability for intellectual leadership.”⁶⁶ Several of the winners of the Stanford competition did not go into mathematics but went on to specialize in electrical engineering (1946), physics (1947), biology (1948) and geology (1956).⁶⁷

It is interesting to note that by introducing Pólya’s article about the 1953 Stanford Competitive Examination, the California Mathematics Council Bulletin found it important to make a connection between “the best interests of democracy” and the need “that our superior students be challenged by courses of appropriate content, encouraged to progress in accordance with their capacities.”⁶⁸ It seems as if the Competitive Examination was viewed by some as reflecting the dangerously mounting international tensions, somewhat forecasting the era of the Sputnik fears yet to come. Speaking at the National Council of Teachers of Mathematics in 1956, Gábor Szegő articulated this opinion when declaring that “much is said in these days about the pressing need for science and engineering graduates. Our view is that the nation needs just as well good humanists, lawyers, economists, and political scientists in its present struggle. This is a view which can be defended, I think, in very strong terms.”⁶⁹ (This ominous reference was dropped from a similar introduction by 1957.⁷⁰)

⁶⁵ H. M. Bacon, “The Stanford University Competitive Examination in Mathematics,” Report at the Meeting of the Mathematical Association of America, University of Washington, Seattle, August 20, 1956. George Pólya Papers, SC 337, 86-036, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁶⁶ H. M. Bacon, “The Stanford University Competitive Examination in Mathematics,” Report at the Meeting of the Mathematical Association of America, University of Washington, Seattle, August 20, 1956. Cf. Gábor Szegő, “The Stanford University Competitive Examination in Mathematics,” 16th Summer Meeting, National Council of Teachers of Mathematics, UCLA, August 21, 1956, p. 2. George Pólya Papers, SC 337, 86-036, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁶⁷ H. M. Bacon, “The Stanford University Competitive Examination in Mathematics,” Report at the Meeting of the Mathematical Association of America, University of Washington, Seattle, August 20, 1956. George Pólya Papers, SC 337, 86-036, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁶⁸ G. Pólya, “The 1953 Stanford Competitive Examination. Problems, Solutions, and Comments,” *California Mathematics Council Bulletin*, May 1953.

⁶⁹ Gábor Szegő, “The Stanford University Competitive Examination in Mathematics,” 16th Summer Meeting, National Council of Teachers of Mathematics, UCLA, August 21, 1956, p. 2. George Pólya Papers, SC 337, 86-036, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁷⁰ G. Pólya, “The 1957 Stanford University Competitive Examination in Mathematics, March 9, 1957,” *California Mathematics Council Bulletin*, 1957, George Pólya Papers, SC 337, 87-034, Box 1, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

Through its long and distinguished tenure the Stanford examination proved to be a pioneer in the discovery of mathematical talent not only in California and the West Coast, but nationally in the United States.⁷¹ To this day, George Pólya is best remembered in the United States as one who introduced European models of competitive educational methods of problem solving in mathematics. He served as one of the several bridges that linked Central European as well as distinctly Hungarian patterns of thinking and reasoning to American achievements in problem solving. The study of Pólya may reveal some of the Central European origins of heuristic thinking in the United States.⁷²

Tibor FRANK – Eötvös Loránd University, Budapest,
Hungary
[Currently: Max-Planck-Institut für Wissenschaftsgeschichte
Endereço: Wilhelmstrasse 44
D- 10117 Berlin, Germany
E-Mail: frank@mpiwg-berlin.mpg.de]

⁷¹ David Gilbarg to George Pólya and Gábor Szegő, April 25, 1966. George Pólya Papers, SC 337, 86-036, Box 2, Department of Special Collections and University Archives, Stanford University Libraries, Stanford, CA.

⁷² The first version of this article was published in Hungary in *Polanyiana*, Vol. 6, No. 2, Winter 1997, pp. 22-37. The author is grateful to the Editor for kindly granting permission to publish it in *Revista Brasileira de História da Matemática*.