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VOLTERRA AND THE JOURNEYS OF FRENCH STUDENTS TO ITALY IN THE 1910S

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Abstract

During his long scientific life, Vito Volterra devoted a lot of energy to the cause of young mathematicians. He helped many of them to find their way in research by providing advice on their work and also by helping them enter the academic community. In the present paper, we are studying the particularly important case of the exchanges of students between Italy and France in which Volterra played a fundamental role. We focus on what may be seen as the genesis of a fruitful tradition, unfortunately brutally interrupted by the surge of the Great War.

Keywords: Mathematics, History, Volterra, France, Italy.

[VOLTERRA E AS VIAGENS DE ESTUDANTES FRANCESES PARA A ITÁLIA NA DÉCADA DE 1910]

Resumo

Durante sua longa vida científica, Vito Volterra dedicou uma grande quantidade de energia à causa de jovens matemáticos. Ele ajudou muitos deles a encontrar o seu caminho na pesquisa, fornecendo aconselhamento sobre seus trabalhos e também ajudando-os a entrar na comunidade acadêmica. No presente trabalho, estamos estudando o caso particularmente importante dos intercâmbios de estudantes entre a Itália e a França, em que Volterra desempenhou um papel fundamental. Nós nos concentramos sobre o que pode ser visto como a gênese de uma tradição fecunda, infelizmente brutalmente interrompida pelo advento da Grande Guerra.

Palavras-chave: Matemática, História, Volterra, França, Itália.

Introduction

In his report (Hadamard 1916) for the posthumous attribution of the Francœur Prize to René Gateaux (1889-1914) who died at the front in October 1914, Jacques Hadamard (1865-1963) recalled Gateaux's one-year research stay in the Italian capital during the academic year 1913-1914. He mentioned that the young man went to Rome in order to learn the methods and the theories of *Monsieur* Volterra. Doing so, commented Hadamard, Gateaux was among those who inaugurated a tradition of journeys of French young mathematicians to Rome whose importance cannot be too much applauded.

In Hadamard's mind, a new kind of scientific cooperation between Paris (or France, more generally) and Rome had been initiated in the years immediately preceding the First World War. The official beginning of the exchanges can be dated from 1912, a year in which a first French student, Joseph Pérès (1890-1962), had sojourned in Rome. A kind of tradition then started and continued until the war. Each year, the mathematician Vito Volterra (1860-1940) was supposed to receive a French student and gave him the possibility of making his first steps in scientific research.

The breaking of the war interrupted this nascent process and put into parentheses most scientific (or at least academic) exchanges between European nations for four years. At first glance, one may think that there was no reason for the exchanges to go on in the 1920s differently from how they had begun in the 1910s. And yet comparing the situation in 1912 to that prevailing, say, in 1925, one soon realizes that the picture had deeply changed. The war and its numerous consequences (human, material, intellectual and so on) had transformed the situation to a large extent. The mathematicians who had been involved in organizing the exchanges, such as Emile Borel (1871-1956) and Volterra were now overwhelmed by new interests. Gateaux was dead, and Pérès, beginning his university career in Toulouse, Strasbourg and Marseilles, was less concerned with Rome though he kept an excellent personal relationship with Volterra. Moreover, what had obviously changed was the Italian political situation which became more tense, caughting Volterra in the vortex very soon after Mussolini's access to power.

We shall therefore focus here mostly on the situation prevailing after 1912 and before the First World War, when was inaugurated the transalpine motion evoked by Hadamard, and several French students followed one another in Rome to accomplish a research program related to the domain Vito Volterra had created some twenty years earlier: Functional calculus.

The aim of the paper is to try to answer several questions which seem historically significant in spite of the brevity of the studied period. Why has this tradition started? Why did studying with Volterra become a natural reflex for these students? Which circumstances had favored the emergence of these journeys at the beginning of the 1910s? Who came and what for?

We shall see that several factors of quite different nature merged to allow this promising collaboration to take place.

In the first place, there had been personal meetings between Vito Volterra and Emile Borel on one hand, and between Vito Volterra and Jacques Hadamard on the other hand.

The proximity of the three men played an important part in the creation of exchanges of students between France and Italy. If Borel, Hadamard and Volterra met for the first time at the occasion of the International congress of mathematicians of 1897 in Zürich, the two relationships followed in fact quite different trajectories, schedules and agendas that we shall need to take into account.

Borel and Volterra seem to have appreciated each other at first sight. Probably, they were soon conscious of the numerous interests they shared: about the conception of scientific activity, about the role played by international cooperation, about the importance of cultural exchanges in the Franco-Italian dialogue… In 1897, they probably did not follow exactly the same agenda, due to their difference of age. Volterra was older by 11 years, and already well known on the international stage, while the ambitious Borel, a recent Doctor and still not getting a permanent position in Paris, was looking for greater visibility. Surely, he envisaged a correspondence on an equal footing with Volterra with some pride. However, their abundant correspondence testifies to the sincerity of their relation and their desire to cooperate for disseminating scientific knowledge. In the future, they would often meet, discuss, and both would unceasingly multiply institutional initiatives in order to facilitate the circulation and meeting of scientists from all over the world.

For Hadamard, the personal acquaintance with Volterra was more gradual and on a more strictly scientific basis. In 1897, Volterra and Hadamard appeared to be merely colleagues working on the same topic - Partial Differential Equations ; only around 1902 when he became conscious of the unexpected importance of Volterra's works on functional calculus had Hadamard begun to be more and more interested in his Italian colleague, and then began a friendship essential for our story.

In a first section, I study the two aforementioned personal relationships. I particularly emphasize how the different natures of these relations, the first one beeing more itimate and involved in institutional contacts, the second more concentrated on mathematical aspects, became a fertile ground for the creation of an exchange process between Paris and Rome. This point is well illustrated by the case of Maurice Fréchet briefly described afterwards ; though he did not come to Italy, Fréchet was advised by Borel and Hadamard to contact Volterra for constructing his mathematical agenda and appears therefore as a genuine precursor of the students of the 1910s.

In a second section, I explain how new financial opportunities allowed to create grants for journeys payed by the university of Paris. This creation was a need underlined by several scholars worried by the lack of mobility of French students in comparison with their fellow students abroad. Finally, a last section is devoted to a more specific description of the students who came to Rome during the few years we consider in this paper.

The two pillars

Volterra's philosophy of progress through science was much in phase with the generous - maybe somewhat naive - ambitions of many scientists of the end of the 19th century and with the Italian patriotic desire of displaying the successes of the prodigious development of mathematics in Italy after the Risorgimento. Very soon in his scientific life,

Volterra had been ready to take his pilgrim's staff in order to use a phenomenal energy to create institutional frameworks to help Italian science to gain a first rate presence on the international stage. He was convinced very soon of the importance not only of going abroad in order to *export* Italian achievements, but also to attract to Italy first rate scientists and also first rate students who would help consolidating the scientific and cultural relations between Italy and its partners in the future. The example of the legendary journey by Betti, Casorati and Brioschi to visit Riemann in 1858 was considered a founder event by Volterra as testified by the subsequent choice of this subject for his general audience conference at Paris International Congress of Mathematicians in 1900 (Volterra, 1900). Many aspects of Volterra's attitude towards science and its role are described in the two biographies published some years ago (Goodstein 2007) and (Guerraggio and Paoloni 2008).

A good occasion for expressing Volterra's conception of scientific relations was given to him by the first International congress of mathematicians in Zürich in 1897. The Italian delegation comprised twenty members, and Volterra soon showed himself one of the most enthusiastic supporter of such meetings. The French delegation was much more limited than one may have expected, also about twenty members, when the Germans were more than the double. Emile Borel and Jacques Hadamard were among the French delegates.

Borel and Volterra

Why was the French delegation so small? Was it because numerous Paris mathematicians, considering Paris as a cardinal point of the mathematical world where anything important eventually arrives, did not see the point of going abroad? Or was it because they had some reticence towards the choice of the place considered as slighlty too German?

Emile Borel wrote a long and spicy review of the Zürich congress when he was back in Paris (Borel, 1897). He criticized his French colleagues who did not consider coming to Zürich for having not understood the importance of such a meeting and noted the disappointment their absence caused among the delegates. Borel insisted on the interest of meeting other mathematicians in person so that the living word could sometimes take the place of cold printed articles ("*pour que la parole vivante prenne de temps en temps le pas sur le froid imprimé*"). And, in Borel's perception, it was a great generosity that the future congress in 1900 was nonetheless attributed to Paris, a real challenge to be taken up.

Borel and Volterra seem to have become fond of each other at Zürich and this marked the beginning of a profound friendship which lasted until Volterra's death in 1940. A huge set of more than 400 letters in the archive of the *Accademia dei Lincei* testifies that Borel most probably became Volterra's closest foreign colleague.

Borel's first letter to Volterra came soon after the congress and exposed his wish to remain in contact with his Italian colleague:

> *If the personal relations which were established in Zürich should faint for three or more years, the biggest and most pleasant advantage of the congresses would be lost.*[1]

From the very beginning of their exchanges, the letters alluded to non professional subjects such as the domestic or foreign policy in both countries. Borel was already oriented along the social-radical tendency he professed during his whole life. Both men appeared concerned with the improvement of the bilateral relation between France and Italy. This relation had considerably deteriorated immediately after the Italian unification in 1870 and remained very tense for almost three decades. The Italians, worried about French possible new claims as a reward for their help during the independence wars (France had already obtained the region of Savoie and the county of Nice) had eventually concluded a military defensive alliance with Germany and Austria. Moreover, at the end of the 1880s, numerous social crises occurred in the south of France involving French and Italian immigrant workers. The slaughter of Italians seasonal workers in 1894 at the salines in Aigues-Mortes, a small city on the Mediterranean sea near Arles, executed by some French workers accusing the Italians of unfair competition gives an idea of the violence of anti-Italian feelings in the France of that time - see (Noiriel 2010). The Italian prime minister of the moment, Francesco Crispi, had adopted an extremely hostile position towards France, as well as towards socalists opposed to his aggressive politics. Crispi's fall after the defeat of the Italians at Adwa during the first Italo-Ethiopian war, marked the beginning of a timid improvement of the relations between both countries.

In one of his first letters to Volterra, Borel commented for instance on Felice Cavalotti's death (1842-1898) as an occasion for both countries to celebrate the memory of this historical founder of Italian radicalism. One may find also in the letters hints of a cultural complicity between Borel and Volterra, for instance when the latter described one of Sarah Bernhardt's performance in Turin.

A noticeable fact is that discussions on mathematics are mostly present in the correspondence only at the beginning of the exchanges. This can probably be explained by the fact that Borel, who was buried in his work on the theory of functions at the turn of the century, was more connected to Volterra's interests of an older period (in particular concerning questions of integration) than with Volterra's subsequent studies on Partial Differential Equations and functional calculus to which the Italian mathematician was then devoting his time.

Borel's attention was soon attracted to Volterra's good will for supporting the nascent career of young students[2] when his young colleague René Baire (1874-1932) had obtained one of the rare grants provided by the University of Paris in order to spend some time in Italy in 1898, especially with Volterra in Turin. Baire was reflecting on questions

[1] Si les relations personnelles qui se sont nouées à Zürich devaient s'éteindre pendant trois ans ou plus, le plus grand et le plus agréable des avantages des congrès serait perdu.(Borel à Volterra, 14 novembre 1897)

[2] Let us mention also in passing that several studies focused in special aspects of Volterrra's broad attention to students. Linguerri, for instance, has studied in her recent book (Linguerri 2011) Volterra's exceptional and constant interest in supporting the careers of young female scientists.

concerning the regularity of functions (the theme of his future doctoral thesis) and he was particularly influenced by Dini and Volterra's studies of discontinuities of real-valued functions of one or several variables. Baire's idea was to define classes of functions depending on their regularity (continuous, limits of sequences of continuous functions, limits of sequences of the former limits and so on). Soon afterwards, Baire's research stimulated Lebesgue's work which led to a revolutionary new notion of integral (see (Hawkins1975), pp.117-118). When Baire defended his thesis in 1899 under the title *On the functions of real variables*, published later as (Baire1901), it bore a dedication to Dini and Volterra.

Baire was only a few years younger than Borel and had been his codisciple at the Ecole Normale. One may therefore be surprised to see the tone of the letter sent by Borel to Volterra in order to recommend the young man; it is closer to the manner of a professor recommending his student than of someone supporting a friend:

> *I have read with the utmost interest what you have communicated to Baire and I am convinced that your enlightened advices will be of the greatest profit for his research. He is besides an intelligent young man in whom you will certainly have pleasure to be interested*[3]*. (Borel to Volterra, 3 mars 1898)*

The young Borel, who still had no official function in the academic institutions, showed himself quite willing to play some institutional role, as his exchanges with Volterra reveal. For instance, Borel helped his colleague organizing the exchange of the *Bulletin de la Société Mathématique de France* with the *Atti dell'Accademia di Torino* ; this was eventually successfully concluded after months of bureaucratic hassle. Also, at the occasion of the preparation of Paris International Congress in 1900 it was Borel who transmitted to Volterra Poincaré's invitation for a general audience conference.

The letters show how Borel's academic position was gradually growing, along with Volterra's own institutional activity in Italy after his installation at the University of Rome in 1901 and his nomination as Senator by the king.

In 1905, as the editor of a collection of volumes dedicated to the theory of functions, Borel asked Volterra about a possible contribution, that he would consider *as an exceptional testimony of Volterra's precious friendship*. The volume in fact never appeared and was probably never written. But the same year saw the foundation of the *Revue du Mois* by Borel and his wife Camille Marbo, and the first paper published in the *Revue* was an article by Volterra, the translation of his inaugural lecture (*prolusione*) given in 1901 at the University of Rome shortly after his nomination there. This important text had a profound influence on Borel, who was beginning to study the laws of randomness and

[3] J'ai lu avec le plus grand intérêt les communications que vous avec faites [à Baire] et je suis assuré que vos conseils éclairés lui seront du plus grand profit pour ses recherches. C'est d'ailleurs un jeune homme intelligent auquel vous aurez certainement plaisir à vous intéresser.

discovered the range of English statistics and new applications of mathematics to biology or economics - see (Israel, 1990) and (Durand and Mazliak, 2011).

Between 1906 and 1910, few letters were exchanged, surely due to the frenetic activity of both scientists. Borel and Volterra met at the Rome International congress of mathematicians in 1908, as is warmly remembered by Camille Marbo in her memoirs. The correspondence resumed again intensely at the beginning of the 1910s as we shall see below.

Hadamard and Volterra

Volterra met Hadamard for the first time also in Zürich in 1897, but their personal relationship really began after their third meeting at Heidelberg's international congress in 1904. The seven years separating these two dates are significant of the mathematical orientation of their relationship. This time interval above all allowed Hadamard to get a better knowledge of Volterra's works. As I cannot give too many technical details in the present paper, I refer the reader to Maz'ya and Shaposhnikova's biography of Hadamard, as well as to the forthcoming more complete study (Guerraggio, Jaeck and Mazliak, 2014) for a better picture.

If Volterra had considered partial differential equations since his first studies in mathematical physics at the very beginning of his career, Hadamard began to look at this topic only around 1894 after the physicist Pierre Duhem (1861-1916) joined him at the university of Bordeaux. Duhem had published an important textbook devoted to hydrodynamics and the theory of elasticity in 1891 where he made use of the most recent results to treat the partial differential equations appearing in these physical theories, such as Riemann's method of characteristics and Kirschhoff's integral representations of solutions. As soon as 1892, some of Duhem's results were improved by Volterra, who began a regular correspondence with Duhem at this occasion. Volterra's works were subsequently extended by other mathematicians such as Orazio Tedone (1870-1962) or Jean-Marie Le Roux (1863-1949). Hadamard mentioned in (Hadamard 1927) how the discussions he had with such a passionate scientist as Duhem had a profound effect on his own scientific orientation.

In Zürich in 1897, Hadamard gave a prospective talk about the benefit one could expect by providing some sets of functions with a (topological) stucture allowing to formulate problems of extrema for solving partial differential equations. Inspired by the calculus of variations and Riemann's treatment of Dirichlet's problem, Hadamard pointed to situations in which the solution may be seen as the function minimizing some functional of energy (his vocabulary was obviously not this one). Hadamard's communication provoked reactions from Salvatore Pincherle who was part of the Italian delegation. Pincherle mentioned that several Italian mathematicians - he quoted Ascoli, Volterra, Arzelà and himself - had been involved in research on sets of functions[4].

[4] The role played in the development of functional analysis by Volterra's works on functional calculus and Pincherle's on complex functions are deeply analyzed in (Siegmund-Schultze, 1982). This author describes in particular how Hadamard's considerations at the Zürich conference were afterwards extended by Fréchet. Other

We do not have any indication about a possible reaction from Hadamard to Pincherle's remark which seems to have remained without immediate consequences. There is only, in a later document, a hint of discussions which may have involved Volterra and Hadamard. In the preface of (Volterra, 1913), Volterra mentioned *private conversations* he had in Zürich about several questions related to functional calculus.

In fact Hadamard, who had just obtained a position in Paris, was quite busy with the preparation of his lectures at the Collège de France, subsequently published as *Lectures on the propagation of waves and the equations of hydrodynamics* (Hermann, 1903). In 1900 occured the second International congress of mathematicians in Paris. Volterra and Hadamard were both present and each gave a conference devoted to partial differential equations in the same session on August 10th. Volterra's communication, soberly entitled *On partial differential equations* mainly recalled results he had published in a paper in 1894; Hadamard's talk, *On partial differential equations with real characteristics*, proposed a brief insight in the comparison between the elliptic and the hyperbolic case for second-order linear partial differential equations. There is no remnants of the discussions between both mathematicians though it is possible that Volterra had given some hints on his theory of functions of lines to Hadamard at this occasion. A few weeks after the conference, Hadamard sent his first letter to Volterra, recently nominated in Rome, but this letter is rather superficial, asking Volterra not to forget to send the text of his conference to the secretary of the congress for the publication in the proceedings. Thus, Volterra and Hadamard, though being in good terms with each other, do not seem to have been already particularly intimate. Hadamard had esteem for his Italian colleague. He gave him a deserved place among the investigators of methods for solving hyperbolic partial differential equations, but he probably did not expect epoch-making results on the topic from him. Besides, in the years between 1900 and 1903, Hadamard was more in contact with another Italian, Tullio Levi-Civita (1873-1941), who had just obtained new results in hydrodynamics - see (Nastasi and Tazzioli, 2006. p.87-89).

The situation was going to evolve before Heidelberg's International congress.

In 1902, Hadamard published (Hadamard, 1902). That was his first work using a formalism where functions are considered as variable elements, five years after the conference in Zürich and Pincherle's observation. In his book on the mathematician's psychology of invention in research process, Hadamard recalls his own surprise when he began to consider functions of lines:

> *Much more surprising is the fate of the extension given to that initial conception [of calculus of variations] in the last part of the nineteenth century, chiefly under the powerful impulse of Volterra. Why was the great Italian geometer led to operate on functions as infinitesimal calculus had operated on numbers, that*

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authors, such as (Archibald and Tazzioli, 2013), study how problems in mathematical physics - particularly those connected to the theory of elasticity - were at the origin of Volterra's studies in functional analysis.

> *is to consider a function as a continuously variable element? Only because he realized that this was a harmonious way of completing the architecture of the mathematical building, just as the architect sees that the building will be better poised by the addition of a new wing. One could already imagine that [...] such a harmonious creation could be of help for solving problems concerning functions considered in the previous fashion; but that "functionals", as we called the new conception, could be in direct relation with reality could not be thought of otherwise than as mere absurdity. Functionals seemed to be an essentially and completely abstract creation of mathematicians. Now, precisely the absurd has happened. Hardly intellegible and conceivable as it seems, in the ideas of contemporary physicists (in the recent theory of "wave mechanics"), the new notion, the treatment of whoch is accessible only to students already familiar with very advanced calculus, is absolutely necessary for the mathematical representation of any physical phenomenon. Any observable element, such as a pressure, a speed, etc., which one used to define a number, can no longer be considered as such but is mathematically represented by a functional! (Hadamard, 1945. p. 129-130)*

In this text written forty years later during the dramatic events of his exile to United States during the Occupation of France, Hamadard slightly embellished Volterra's motivations for considering functions of lines. Those were not exactly the ethereal aesthetic aim presented by Hadamard. As soon as his first 1887 paper on the topic (Volterra, 1887), the Italian mathematician emphasized how many problems of Mechanics and Physics naturally make use of quantities depending on all the values assumed by such or such parameter, as in the instance of the temperature at a point of a conducting strip which depends on the temperature at every point of the edge.

In the aforementioned 1902 article, Hadamard merely studied and slightly extended the notion of derivative of a function of line Volterra had previously introduced. But it is mainly in a 1903 note (Hadamard, 1903) that he began to present how the notion of function of line, extended to that of function of a surface, could be used to obtain a differential equation for functions appearing in some physical problems, such as the Green function associated with two points interior to a surface S whose edge is deformed. Hadamard became thus conscious of being on a ground where he could meet, or even challenge Volterra. The situation slightly embarrassed him and one may notice a touch of concern in the letter he sent to his colleague a few weeks before the Heidelberg's conference:

> *I have always forgotten to ask you about what you intend to present at Heidelberg congress (section of applied mathematics). May I ask you to give me some information on that point, so that*

I would not compete with you on your own ground[5].

Volterra's generous answer sent at the eve of the conference probably comforted Hadamard:

'I thank you very much for your kind letter. I intend to say something about the theory of waves at the congress in Heidelberg. The subject is so extended that I am sure that if you want to talk about the same subject, there would not be only interferences between both communications. (…) I certainly shall not forget to quote your note to the Société Mathématique de France from 1903 where you use the method of waves. If I have time I would like to evoke the relation between vibrations of membranes and the theory of waves. Maybe the program is too extended. If you would like to make some comments, I would be very obliged to you'[6].

Volterra and Hadamard met in Heidelberg in August and presided a session together on August 10th, where, apart from them both, A.Sommerfeld and R.W.Genese gave talks (Krazer 1905, p.46). Hadamard made in fact two conferences in Heidelberg. In the other one, entitled *On the fundamental solutions of linear partial differential equations*, Volterra is widely quoted as the main source of the theory. From that moment, Hadamard clearly became the French mathematician whose interests were the closest to Volterra's, and their later letters show a growing sympathy between the two men.

A first instance of a 'studental coproduction': Maurice Fréchet

It is not that surprising that the friendship and the common scientific vision shared by Volterra and his French colleagues led Borel and Hadamard to advise young students to get in touch with the Italian mathematician. The case of the young Maurice Fréchet offers a good instance of these joint efforts. Maurice Fréchet (1878-1973) had just finished the Ecole Normale and was Hadamard's protégé since his school days. Taylor (Taylor, 1982, p. 242-3) mentions how Hadamard told Fréchet he had seen Volterra in Heidelberg who told him he was willing to receive material that Fréchet might send him.

Fréchet contacted Volterra, advised also to do so by Borel. He sent a first letter at the beginning of 1904. It is an extremely long letter in which Fréchet exposed his projects for developing a differential calculus for the functions of lines. He explained that he had been in charge of the edition of the just published latest lectures both of Hadamard and

[5] J'ai toujours oublié de vous demander ce que vous avez l'intention de traiter au Congrès de Heidelberg (Section des mathématiques appliquées). Puis-je vous demander de me renseigner sur ce point, afin que je n'aille point sur vos brisées? (Hadamard à Volterra, Juillet 1904)

[6] Je vous remercie beaucoup de votre aimable lettre. J'ai l'intention de dire quelques mots au congrès de Heidelberg sur la théorie des ondes. Le sujet est si vaste que je suis sûr que si vous voulez parler aussi du même sujet il n'y aura pas que des interférences entre les deux communications. (…) Je ne manquerai pas de noter à ce propos votre note de la Société math. de France de 1903 où vous employez la méthode des ondes. Si j'aurai le temps je voudrais toucher à une relation entre les vibrations des membranes et la théorie des ondes. Peut être le programme est trop vaste. Si vous voulez bien me faire quelques remarques je vous en serai fort obligé. (Volterra à Hadamard, 27 juillet 1904)

Borel, respectively on Calculus of Variations and Series of polynomials, and now wrote to Volterra in order to choose a theme for his doctoral thesis. Still a student at the Ecole Normale, after having attended some Hadamard's lectures at the Collège de France where Hadamard mentioned Volterra's theory, Fréchet had obtained some small results about the variation of some functions of lines and he would have liked to know Volterra's opinion on them and how to extend them. At the end of the letter, Fréchet wrote a noticeable comment about Volterra's reputation among French young mathematicians:

> *I deeply pray you to forgive me to have disturbed you for so long; I would certainly not have taken this liberty if Monsieur Borel had not spoken in so flattering terms of your benevolence for young students. And I obviously do not speak about the high esteem of the French mathematicians for you*[7].

Volterra advised Fréchet to publish his results - they were published after several months in the *Annales de l'Ecole Normale Supérieure* as (Fréchet, 1904a) - and also suggested him to work on an extension of the Hamilton-Jacobi theory. During the following summer, Fréchet sent a paper on that topic which Volterra managed to publish in the *Annali di Matematica*. In a subsequent letter of December 1904, Fréchet mentioned that he had obtained some new theoretical results but admits that he was somehow overcome by doubt about the possible applications of *functional calculus to Physics or to Geometry though they are probably numerous*. Such a sentence may be an indication that Fréchet was gradually shifting his thesis topic to a more topological approach, partly in relation with Hadamard's suggestion in Zürich, as is suggested by Taylor (Taylor, 1982 p.259) in his extensive study of Fréchet.

The new opportunities for travels

We shall now focus on an evolution of the academic situation that occured around 1910 and facilitated the organization of travels of French students to Italy.

At the beginning of the 20th century, several academics were conscious that the French student remained too much shut inside the borders of the country, and even inside the limits of his own university, absolutely ignorant of how things were going in the outer world. The biologist Maurice Caullery[8] published a text in 1907 in Borel's *Revue du Mois* where he emphasized the problem (Caullery 1907).

[7] Je vous demande mille pardons de vous avoir importuné aussi longtemps; je ne me le serais certainement pas permis si Monsieur Borel ne m'avait pas parlé en termes aussi flatteurs de votre bienveillance pour les jeunes étudiants. Je ne parle pas, bien entendu, de la haute estime en laquelle vous tiennent les mathématiciens français. (Fréchet à Volterra, 1904)

[8] Caullery was in 1917 the author of an intersting study (Caullery, 1917) about scientific life in United States at the beginning of the 20th century.

> *One of the scourges of our university customs is the absence of circulation between the different universities. The migration of young students or even young professors is a strength of Germany. In France, the student remains clinched to the city where he began his studies as the parasit to his host, unless he comes to increase the congestion in Paris*[9].

As seen in the previous quotation, the backdrop of the problem was the competition with Germany, some decades after the humiliating defeat of 1870 in the Franco-Prussian war. This problem has been adressed by Christophe Charle in several studies - see in particular (Charle, 2003). In the paper mentioned earlier, Caullery mentions the great impression made by German scientific organization on a young Japanese scientist who called Germany *the new world in the old*. For Caullery, this was an urgent problem to fix because the scientific journeys are an occasion of creating a cultural and political proximity, especially because the German influence was growing in the United States.

> *The growing number of young scientists who visit Germany influence already the American opinion. Berlin*[10] *begins to be considered as the world scientific Mecca. There is, in several circles in America, an enthusiasm for the German way of thinking which comes unconsciously along with a sympathy towards the political aspirations of Germany. This is this kind of causes which often produce national sympathies or antipathies. Americans studying in Europe are those who will educate the growing generation, and the sympathies of the American nation will be guided by these students*[11].

The American mathematician James Pierpoint (1866-1938) wrote a report for the AMS in 1899 in order to encourage American students in mathematics to try to go to Paris. Looking attentively to the comparison between German and French situations reveals the residual rigidities of French educational system

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[9] L'une des plaies de nos mœurs universitaires est l'absence de circulation entre les différentes universités. C'est une des forces de l'Allemagne que les migrations régulières des étudiants et même des jeunes professeurs. En France, l'étudiant reste rivé à la ville où il a commencé ses études comme le parasite à son hôte, à moins qu'il ne vienne augmenter la congestion de Paris.

[10] One of the referees suggested that Berlin is mentioned here because the author is a biologist - a mathematician would have probably written Göttingen instead. In fact, in my opinion, Berlin is here above all taken as a personification of Germany.

[11] Les jeunes savants qui visitent l'Allemagne en nombre croissant influencent déjà l'opinion américaine. Berlin commence à être considérée comme la Mecque scientifique du monde; il y a, dans certains cercles d'Amérique, un enthousiasme pour la pensée germanique où s'unit inconsciemment une sympathie pour les aspirations politiques de l'Allemagne. Ce sont des causes comme celles-là qui souvent produisent les sympathies ou les antipathies nationales. Les Américains étudiant en Europe sont ceux qui formeront les générations qui grandissent, et les sympathies de la nation américaine seront guidées par ces étudiants.

> *Before leaving the subject of mathematical instruction let me note the difference between the attitude of mind of the French and the German student while preparing for his career as a mathematician. The German university requires 8 semesters to prepare for the doctorate. As there are no examinations till the close, the German student is free to roam about not only in mathematics but in physics, philosophy, history, etc. In fact a part of the rigorosum requires a knowledge of philosophic studies. At the close, the examinations, which are oral, aim not so much at specific encyclopaedic knowledge as a largeness of view and maturity of judgment. (...) In France, on the other hand, from the moment the élève leaves the classe de rhétorique, i.e., from 16 years onward, he has an examination before him at every turn. The plans of his studies are very largely marked out for him ; care is constantly taken to see he makes no lost steps and that he is properly prepared to pass his examinations. The university and the Ecole Normale make the impression of a great institution wonderfully arranged to turn out a certain product, in a certain amount, in a certain degree of excellence, with the least loss of time and energy. (...) Indeed, can we rightfully say more than that each seems best adapted to the needs and temperament of its people? (Pierpoint 1900), pp.242-243.*

This impression of power emerging from the German universities was besides observed also by some young French students themselves. When the mathematician Maurice Janet came to Göttingen in 1912, he wrote on his diary (Mazliak 2013):

> *The Sorbonne, in comparison with Göttingen Universität, gives the slight impression of a fairground. The Ecole Normale - at least the library - seems just to be ruins*[12].

And though, Maurice Janet added, French science had the same, or even better, successes than the German one.

In 1913 was published in France an enquiry (Agathon 1913) about French students written by two journalists, H.Massis and A. de Tarde, who chose the pen name Agathon, entitled *Young people of today*. Despite its primitive character, it was one of the first modern poll in France, as the authors proposed questionnaires to hundreds of students and tried to make a synthesis. The publication was deeply influenced by the proximity of the authors to the ultra-nationalist *Action Française*, and its results are also biased by the fact

[12] La Sorbonne, à côté de Göttingen Universität, donne un peu l'impression de "Foire sur la Place". L'Ecole - au moins la bibliothèque - donne celle de "ruines".

that the questionnaires had more or less been distributed only to a sample of quite privileged students of Paris *Quartier Latin*. However, the book offers important insights into the state of mind of the golden French youth on the eve of First World War ; among them, the revelation of a true etnhousiasm for traveling. Agathon gives an unexpected comment on this desire:

> *'The habit of travels, far from weakening the idea of fatherland, has transformed and precised it. Those who travel feel better the opposition between themselves and strangers; they become more conscious of the differences. Anytime I have been abroad, declared a young student in literature to us, I have experienced the truth and the strength of patriotic feelings in me* [13]

According to Agathon, these travels helped ascertain the superiority of France with respect to its competitors, a conclusion that perfectly fit the author's nationalistic agenda. This was a clear and intentional overinterpretation of the results of the poll. But anyway the new generation seemed more open to the world than its predecessors.

Around 1910 an important change occured in the perspectives offered to students at the University of Paris. Two major legacies were given to the institution by rich sponsors in order to create a system of grants attributed to selected students in order to complete their studies and often to achieve a doctorate. The Commercy legacy to the Faculty of Sciences began in 1908, and the total amount of money was devided among the different disciplines. Mathematicians were provided with a budget of 30000 francs per year to be attributed to students chosen by a commission of professors of the University in which Emile Borel played a major role in the 1910s. A new legacy of 150000 francs for five years then was given in 1909 by the businessman David David-Weill (1871-1952). This one is more significant for our story in that it was explicitely devoted to the creation of grants for travels abroad of students in all disciplines whereas the Commercy grants were attributed on the more indistinct purpose of contributing to scientific work. A noticeable requirement of the David-Weill's foundation was that the candidates to the grants were not supposed to provide a research program for their stay because the donator's aim was more to offer future teachers a possibility of a better knowledge of a foreign country than to encourage them to follow a research program. It was expected that those students would be in position to enlighten their pupils with the observations they had accumulated abroad when they would become teachers themselves. David David-Weill at the beginning had thought of short stays of three months but the rector convinced him that longer stays would be more profitable. It was therefore decided to attribute ten grants of 3000 francs each year to selected students, both male or female ; generally, it was required from them to have passed

[13] L'habitude des voyages, enfin, loin d'affaiblir l'idée de patrie, l'a transformée et précisée. Ceux qui voyagent sentent le mieux l'opposition des étrangers à eux-mêmes: ils prennent conscience de leurs différences: "Chaque fois que je me suis trouvé à l'étranger, nous déclarait un jeune étudiant de lettres, j'ai éprouvé en moi la vérité et la force du sentiment patriotique."

the competitive examination of Agrégation. The amount of 3000 francs was quite generous in 1910 (we shall see below that it would not be the same in the post-war situation). Let us recall for instance that the mean salary of a factory worker was then around 4 francs per day (and sometimes much lower). At the beginning, the commission for the attribution of grants was presided by Emile Durkheim and the beneficiaries were exclusively students of humanities (linguists, historians, philosophers and so on). Some of them may have had real scientific interests as it was the case with the young student in philosophy Maurice Halbwachs (a future prominent sociologist) who belonged to the first group obtaining a David-Weill grant and went to Vienna and Berlin. Another partial explanation for this absence of students in science among the first beneficiaries of the David-Weill grants relates to a very different problem. An unexpected difficulty occured when the rector realized that he had the military service to take into account. Since the 1905 law, all the young men were required to give two years to the military - including those who had gone to the Grandes Ecoles which were largely exempted before. In these years, the Ecole Normale was calling the shots in the scientific disciplines and Jules Tannery (1848-1910), vice-director of the Ecole Normale for the sciences (whose position was given to Emile Borel after Tannery's sudden death in December 1910) warned the minister of Education that no scientific students of the Ecole Normale would be immediately susceptible to benefit from a grant before 1912. This probably postponed the consideration for the grants to students of scientific disciplines during the first two years.

French students go to Volterra

The grants slowly became accessible in favour of mathematicians from 1912 on, and precisely to go to Rome in order to study with Volterra. Before describing the details of these few travels, I would like to propose some kind of explanation why Volterra and Rome have been considered at that moment as adequate destinations for young students.

In 1907, Hadamard had completed the variational approach to the physical phenomenon of vibrations he roughly sketched in his 1903 note. He composed the long paper *Memoir on the analytical problem of equilibrium of fitting elastic plates* (Hadamard, 1907) crowned by the Prix Vaillant of Paris Academy of Science. In 1909, Hadamard had been chosen for a tenure at the Collège de France and this was soon followed in 1912 by his election to Paris Academy of Sciences. He devoted his course at the Collège de France to problems of mathematical physics and commented on Volterra's and his own functional approach.

One finds there the origin of Paul Lévy's research on potential theory, giving another instance of the presence of Volterra on the Parisian mathematical stage of the time. Attending Hadamard's lectures in 1910, Paul Lévy submitted to him the idea of choosing the systematic study of functional equations as the subject for his thesis. In his autobiography, (Lévy 1970. p.42), Lévy writes that Hadamard replied: *As you have taken the subject in hands, I leave it to you*. Some months later (November 1910), Lévy sent a first letter to Volterra and recommended himself on behalf of his father Lucien Lévy - a

professor of mathematics whom Volterra had met when he was in Paris - and above all of Hadamard. The motivation for the letter was to ask Volterra to send recent papers on functional calculus because, being professor at the Ecole des Mines in the city of Saint-Etienne, it was difficult for Lévy to have access to scientific literature. Volterra sent him a few documents. Lévy defended his thesis the year after. Hadamard, who saw Lévy as his natural successor in the studies in potential theory wrote to Volterra an enthusiastic comment on Lévy's work:

> *'Have you already looked at Paul Lévy's thesis? In the opposite case, I think it would interest you (I shall ask him to send it to you). No doubt you will consider with me that it brings a first rate contribution to the beautiful theories of functional calculus that you have provided and on which I tried to work after you*[14].

Joseph Pérès

In 1911, Borel invited Volterra to read lectures at the Sorbonne about functions of lines and integro-differential equations. Their correspondence does not really reveals why this theme was selected. A possible hypothesis may be found in Hadamard's interests we have just mentioned and Hadamard may have whispered the topic to Borel. Before Volterra's arrival, Borel proposed to him that one of his and Hadamard's most promising young students of the Ecole Normale, Joseph Pérès, would take care of the edition of these lectures for their publication. Pérès gladly accepted the task and probably at this occasion was born in Borel - or in Pérès, or in Hadamard, or in the three of them - the idea of proposing Pérès for a David-Weill grant. Pérès immediately wrote to Volterra to confirm his wish to go to Rome. Volterra expressed his interest for receiving visiting students from Paris and sent a letter of support of Pérès' demand to Louis Liard, vice-rector of Paris university. Moreover, he also told Pérès to have selected some themes of particular interest for him as subject for the lectures Volterra was to give in Rome during his stay, a sign of a noticeable attention towards the protégé of his friend Borel. On 4 June 1912, Pérès informed Volterra of the acceptance of his grant. During the following summer, he finished the edition of Volterra's lectures on functions of lines: the book was published during the fall of the same year.

Pérès arrived in Rome at the beginning of November 1912[15]. He seems to have worked with energy in the Italian capital and published several notes in the *Rendiconti of the Accademia dei Lincei*.

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[14] *Avez vous déjà parcouru la thèse de Paul Lévy? Dans le cas contraire, je pense qu'elle vous intéressera (je vais lui demander de vous l'envoyer). Vous estimerez sans doute avec moi qu'elle apporte aux belles théories de Calcul fonctionnel que vous avez émises et dont j'ai essayé de m'occuper après vous, une contribution de premier ordre.* (Hadamard à Volterra, 1911). Hadamard always had a great esteem for Lévy's thesis (see in particular how he mentioned it in his conference at the 1928 Bologna international congress). For details on Lévy's works on potential analysis, one may consult (Barbut, Locker and Mazliak, 2013).

[15] Pérès was not Volterra's only foreign student at that moment. In Rome he met the young American mathematician Griffith C. Evans (1887-1973) who came soon after Volterra's stay in Clark University in 1909 (on

In December, Borel wrote to Volterra to express his satisfaction:

> *I am very glad that you are satisfied of M.Pérès' work and I rejoice for him that you are interested in his research*[16].

The main proof of the success of this first journey is seen in the lines sent by Volterra to Liard, in a short report on Pérès:

> *M.Pérès has much successfully dealt with original research. He published three papers on integral equations and on permutable functions, and he has just approached a new very interesting question about analytical permutable functions. I can ensure you also from my colleagues of the faculty that we are happy to have had in our University a young mathematician as distinguished as M.Pérès and we hope that his career in mathematics will meet the best success*[17].

Back in France, Pérès was candidate to, and obtained, a Commercy grant for the Academic year 1913-1914. In fact, from 1913, not only the lists of candidates for the Commercy and the David-Weill's grants have many names in common - which is natural - but also one may get the impression that the attribution of a Commercy grant *after* a David-Weill grant was considered as a logical process. The idea may have been first to facilitate the travels of young students to open their mind and their perspective by offering a David-Weill grant, and then to allow the most worthy to go on with their scientific work with the Commercy grant. That one in particular did not require any kind of travel: typically, Pérès stayed in Paris the year he obtained the grant.

René Gateaux

In his letter containing the report on Pérès, Volterra declares his desire that another student, René Gateaux, who had expressed his wish to follow Pérès' path to Rome would also be in position to get a grant to do so. The case of Gateaux is thoroughly studied in the

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that subject, see for instance (Guerraggio e Paoloni 2008), Chapter 5). We shall not follow the relation between Evans, Pérès and Volterra in the present paper, though is was a curious mixture of complicity and competition during several years, in particular during the First World War - see (Mazliak and Tazzioli, 2009), Chapter 7) - and would certainly deserve a separate study.

[16] Je suis très heureux que vous soyez satisfait du travail de M.Pérès et je me réjouis pour lui que vous vous intéressiez à ses recherches. (Borel à Volterra, 30 décembre 1912)

[17] M.Pérès s'est occupé avec beaucoup de succès de recherches originales. Il a publié trois travaux sur les équations ntégrales et sur les fonctions permutables, et il vient d'aborder une nouvelle question très intéressante sur les fonctions permutables analytiques. Je puis vous assurer de la part aussi de mes collègues de la faculté que nous sommes heureux d'avoir eu dans notre Université un jeune mathématicien si distingué que M.Pérès et que nous espérons que sa carrière dans les mathématiques aura le meilleur succès. (Volterra à Liard, 24 avril 1913)

paper (Mazliak 2011), to which I refer the interested reader for more details on this singular young man. I shall only briefly consider him in the present article.

Gateaux with the support of Borel and Volterra obtained the David-Weill grant. Borel had written to Volterra in April 1913 to mention the fact:

> *Another young man, who is also my former student, presently professor at the Lycée of Bar-le-Duc, has recently mentioned his intention to ask for a study grant in order to follow research connected to your works. I advised him to ask for a David Weill grant as Pérès and to go to Rome if you agree to welcome him*[18].

Borel forwarded Gateaux's programatic letter to Volterra. Gateaux exposed his desire to work in order to obtain a notion of analytical functional for which an integral representation of the Cauchy type would be valid.

About Gateaux's stay in Rome, I do not have many details. He lived at the same address where Pérès had at 72 Corso Vittorio Emanuele, in the center of the city. An interesting document, found at Paris Academy, is the draft of a report written by Gateaux at the end of his stay for the David Weill foundation. He mentioned that he had arrived in Rome in the last days of October, and that he followed two of Volterra's courses in Rome (one in Mathematical Physics, the other about application of functional calculus to Mechanics). In this report, in a rather touching way, he also described how the grant allowed him to travel around Italy and to educate himself and he insisted how desirable it would be that the French knew modern Italy and Italian language at the same level than Italians did know France and French language.

Gateaux seems to have worked well in Rome. A first note to the *Accademia dei Lincei*, where he extended the results he presented in a previous note to Paris Academy, was published in December 1913. On a postcard sent by Borel to Volterra on 1 January 1914, Borel mentioned how he was glad to learn that Volterra was satisfied with Gateaux. The young man published three more notes during his stay, but also began to write more detailed articles - found after the war amongst his papers. On 14 February 1914, Gateaux made a presentation to Volterra's seminar, in which he mainly dealt with the notion of the functional differentiation. He recalled that Volterra introduced this notion to study problems including hereditary phenomena, but also that it was used by others (Hadamard and Paul Lévy) to study some problems of mathematical physics - such as the equilibrium problem of fitted elastic plates - through the resolution of equations with functional derivatives.

Gateaux came back to France at the beginning of the summer, in June 1914. He expected to go back soon to Rome as he was almost certain, as Borel had written to Volterra, to obtain the Commercy grant he had applied for. Gateaux soon wrote that the

[18] Un autre jeune homme qui est aussi mon ancien élève, M.Gateaux, actuellement professeur au Lycée de Bar-le-Duc, m'a parlé récemment de ses intentions de demander une bourse d'étude en vue de recherches qui se rattachent à vos travaux. Je lui ai conseillé de demander comme Pérès une bourse David Weill et d'aller à Rome, si vous voulez bien l'accueillir. (Borel à Volterra, 18 avril 1913)

grant was accepted.

Jacques Soula

The successor of Gateaux for a David-Weill grant should have been Jacques Soula (1890-1970). Soula is an interesting case as, contrary to his predecessors, he was not a student of the Ecole Normale Supérieure - and not even of one of the *Grandes Ecoles*, a fact which on the French mathematical stage of the time generally implied that you must deploy a wealth of energy to see your application seriously considered. Professor at the Lycée of Aix-en-Provence since 1909, Soula spontaneously contacted Volterra in August 1912, following (so he wrote to the Italian mathematician) the advice of one of his former professors of the University of Montpellier. I have not been able to definitely understand who this professor was. A reasonable hypothesis may be Samuel Lattès (1873-1918)[19]. Anyway, Soula, in his first letter, submitted some results about permutable functions of the second kind, results subsequently published in two notes to the Lincei in October 1912 and February 1913. Soula tried to take advantage of the fact and applied for a grant. Hadamard was along with Borel a referee for the attribution of grants. In May 1913, he wrote to Volterra:

> *M.Soula, professor at the Lycée of Aix-en-Provence, of whom you have presented two notes to the Academy of Lincei (in October 1912 and February 1913, if I am not wrong) is candidate to a grant of Paris University. May I ask you your impression about him and about the works he submitted to you?*[20]

Soula's attempt was not successful. Borel explained to Volterra:

> *As I have written to you, Gateaux has obtained a grant and intends to go to Rome; it has not been possible to attribute a grant to M.Soula this year. Maybe it would be possible next year?*[21]

[19] Lattès, entered the Ecole Normale Supérieure in 1892. He defended his thesis in 1906 and became soon professor at the university of Montpellier. Many details may be found about Lattès in [Audin2011] (see in particular Chapter II). He had published his thesis about iteration in 1906 in Dini's journal, the *Annali di Matematica*. In a letter sent in the 1920s, Soula asked Volterra about the possibility of publishing his own thesis in an Italian journal as *it had been the case for Lattès*.

[20] M.Soula, professeur au lycée d'Aix, dont vous avez présenté deux Notes à l'Académie des Lincei (en octobre 1912 et en février 1913, si je ne me trompe) est candidat à une bourse à l'Université de Paris. Puis-je vous demander de me dire votre impression sur lui même et les travaux qu'il vous a soumis? (Hadamard à Volterra, 21 mai 1913)

[21] Comme je vous l'ai écrit, Gateaux a une bourse et compte aller à Rome; il n'a pas été possible d'en attribuer une cette année à M.Soula. Peut-être pourra-t-on le faire l'année prochaine?

In fact, the 1913 report of the commission for the Commercy grants, written by Elie Cartan, displays a real embarrassment to deal with this outsider:

> *M.Soula is an isolated worker; he has only his salary as teacher to earn a living and to support his family; for these reasons, the subcommission had a sympathetic a priori impression towards his application. However, it has judged that M.Soula's scientific production was too thin to get a precise opinion on his value; it thus proposes to postpone any decision about him; it expresses the hope that in the meantime M.Soula will be in position to justify the good opinion of him expressed by MM. Lattès, Volterra and Hadamard*[22].

Apart from the fact that he did not not belong to the inner circle of the Ecole Normale, it is not absolutely clear why Soula's scientific dossier, containing already publications and a warm support letter by Volterra was considered insufficiently rich to the commission. But Soula was lucky, as Volterra did not forget the vague promise made by Borel. On December 1913, when he wrote to Borel to express his satisfaction about Gateaux, he reminded that Soula had still the desire to come to Rome. Borel's answer did not display a real enthousiasm:

> *For M.Soula, the difficulty I foresee is that, having some family to support, he may not be able to content himself with the David Weill grant obtained by Pérès and Gateaux ; he would therefore need a more important grant. But, naturally, if he seems to you as interesting as the other two, I will do my best to bypass the difficulty*[23].

Borel eventually advised Soula to ask for a David-Weill grant. Soula wrote to Volterra:

> *I received from M.Borel the advice to ask for a grant for traveling to foreign universities from M.David-Weill's foundation and to go to Rome. I am quite prepared to follow this advice: I*

[22] M.Soula est un travailleur isolé; il n'a que son traitement de professeur pour le faire vivre et lui permettre de subvenir à ses charges de famille; pour ces raisons sa candidature était *a priori* sympathique à la sous-commission. Mais celle-ci a estimé que la production scientifique de M.Soula était trop mince pour qu'elle pût se faire une opinion précise sur sa valeur ; elle propose donc d'ajourner à l'an prochain toute décision à son égard; elle espère que d'ici-là M.Soula saura justifier la bonne opinion qu'ont de lui MM.Lattès, Volterra et Hadamard.

[23] Pour M.Soula, la difficulté que j'aperçois est que, ayant de la famille, il ne pourra peut-être pas se contenter de la bourse Weill qu'ont eue Pérès et Gateaux; il faudrait donc une bourse plus importante. Mais, bien entendu, s'il vous paraît aussi digne d'intérêt que les deux autres, je ferai tout mon possible pour lever cette difficulté. (Borel à Volterra, 1er janvier 1914)

> *will certainly profit a lot from your teaching and a sojourn in your beautiful country is not to displease me*[24].

A David-Weill grant was indeed attributed to Soula for the Academic year 1914-1915, as he himself wrote to Volterra soon after, expressing his hope that the administration of public instruction would not make problems for his leave.

The other way round?

A natural question can be raised about the symmetry of the exchanges. If his Parisian colleagues were glad to send students to Rome, Volterra was in a good position to ask them to welcome Italian students to Paris. In fact, I have found just one instance of this. In 1913, Volterra proposed to Borel to send to Paris an Italian student, the young astronomer Giuseppe Armellini (1887-1958). Borel answered that he would be very glad to see and help Armellini in Paris; nevertheless he advised Volterra to give Pérès' address to him because, wrote Borel on 25 August 1913, *young people better understand each other*.

Armellini arrived in Paris during fall 1913 and stayed there the entire academic year. He received a warm welcome and wrote to Volterra regularly to describe his meetings and his works:

> *My welcoming by Borel and Andoyer was perfect. Tomorrow I shall go to Appell, Picard, Hadamard and Lebon. Here at the Sorbonne I am listening to the lectures by Andoyer, Darboux and Borel (Hadamard and Brillouin still do not have begun their lectures at the Collège de France), and I am getting down to my work over the secular shift of lunar libration. I hope to take profit of this year of studies and with the utmost urgence I feel the duty of sending my best thanks and to express to you my unlimited gratitude for all you have done for me from the moment when I had the luck to become your student*[25].

Why did Volterra not try to send more students to Paris? Two main explanations can be proposed to what seems, at first glance, a surprising fact. The first one is that Volterra may not have had in Rome a breeding ground such as the Ecole Normale Supérieure in

[24] J'ai reçu de M.Borel le conseil de demander une bourse de séjour dans les universités étrangères de la fondation de M. David Weill et de venir à l'Université de Rome. Je suis assez disposé à suivre ce conseil: je retirerai certainement grand profit de votre enseignement et de vos conseils et un séjour dans votre beau pays n'est pas pour me déplaire. (Soula à Volterra, 2 avril 1914)

[25] Cosi pure ottima è stata l'accoglianza fattami da Borel e Andoyer. Domani andro da Appel, Picard, Hadamard e Lebon. Qui frequento alla Sorbonne i corsi di Andoyer, Darboux e Borel (Hadamard e Brillouin non hanno ancora cominciato le loro lezioni al "Collège de France"), ed attendo ad un mio lavoro sullo smorzamento secolare del moto di librazione lunare. Spero di trarre profitto di quest'anno di studio e sento intanto vivissimo il dovere di porgerLe i più caldi ringraziamenti, e di significarLe la mia imperitura gratitudine per tutto cio che ha fatto per me, fin da quando ebbi la fortuna di divenire Suo allievo. (Novembre 1913)

Paris to select promising future scientists. The second one may be more important ; the natural destination abroad for Italian mathematicians was probably more Germany than France. In Armellini's case, the original intention of the young man has been to divide his stay abroad between Paris and Berlin. However, Armellini was so satisfied in Paris that he decided to remain there. He wrote to Volterra:

> *The present ministerial decree for my studies plans that I should stay four months in Paris and then four in Berlin; but I hope to be able to obtain its modification so that I could remain in Paris until the end of July. I would therefore have the advantage to follow lectures by Appell and Picard and to stay longer with these professors who have welcomed me with such benevolence*[26].

Armellini remained indeed in Paris until the summer 1914. This gave him the opportunity to attend to the beginnings of what became in the 1920s the Hadamard seminar. He described it to Volterra:

> *Professor Hadamard, as he told me himself last time I have visited him, from time to time will ask students to sum up some of the most important published works of the last years. He desires that these surveys be rather short, not exceeding 15 or 20 minutes for instance so that it would be possible to present four works in each seance. After every survey, the audience would ask for enlightenments or raise objections to which the person who had presented the talk would respond* [27].

The war

The breaking of the First World War in August 1914 annihilated the organization of the expected travels. Pérès and Gateaux were immediately called to the Army. Soula decided to voluntarily engage in the conflict in September 1914 (he was not mobilized immediately due to his dependent family).

[26] Per ora il decreto ministeriale per il mio perfezionamento stabilisce un soggiorno di quattro mesi a Parigi e quattro a Berlino; ma io spero di riuscire a farlo modificare in modo di rimanere a Parigi fino al mese di luglio. Avrei cosi il vantaggio di seguire le lezioni di Appel e Picard e di restare più a lungo in mezzo a questi Proff.ri che mi hanno accolto con molta bonta. (Dicembre 1913)

[27] Il Prof.re Hadamard, come Egli stesso mi disse l'ultima volta che andai a visitarlo, fara riassumere di volta in volta agli allievi alcune delle piu importanti memorie pubblicate negli ultimi anni. Egli desidera che questi riassunti siamo assai brevi, non superiori p.es. ai 15 o 20 minuti ogniuno, in modo da poter trattare quattro memorie per seduta. Dopo ogni riassunto gli uditori domanderanno degli schiarimenti o faranno delle difficoltà, alle quali risponderà colui che ha tenuto la conferenza. (Marzo 1914).

Italy proclaimed its neutrality in August 1914 and this situation prevailed until May 1915. From one of Armellini's letter to Volterra, we observe that the beginning of the war in France did not seem to worry him a lot:

> *Allow me, dear Professor, to wish you a pleasant time of vacation. I have written some times ago to M.Pérès, but he had still not answered to me; probably he also had been mobilized*[28].

Due to health problem, Pérès was in fact soon demobilized and sent back to civil life, as teacher in Montpellier where he remained during the whole war. He was a strange exception during these years of tragedy: during the conflict, Pérès was doing mathematics and, contrary to most of the young students of the same age, did not lose time for his career's advancement. He defended his thesis during the war, and received his first university position immediately in 1919 (see Mazliak and Tazzioli, 2009, Chapter 7).

Gateaux had not this fortune: he was killed on 3 October 1914 near Arras during the *race to the sea*, the dramatic and bloody weeks when the German and the Franco-British armies tried to be the first to reach the Channel. Gateaux's last letter to Volterra is an amazing document, sent from the battle field at the end of August 1914, expressing a deep satisfaction for the Italian decision of neutrality. As for Soula, we possess two letters sent from the front to Volterra in 1915. One, dated from February 12th, expresses the deep wish of an engagement of Italy into the war, in the name of the Latin and Provençal solidarity between the south of France and its neighbour. And it is in the name of the same solidarity that Soula sent his warmest congratulations on July 8th about the Italian decision of entering the war.

The students were not the only men involved in the battle. In 1915, Borel who was then 44 decided to engage himself in the army. So did Volterra, who was 55, after having been one of the leaders of a campaign claiming for Italy's engagement during the 9 months of Italian neutrality. Both went to the front, and took in charge technical aspects of the war: Borel conducted experiments with equipments for the acoustic location of guns, and Volterra was embarked in a large program of development of dirigibles. But both men had also an important political and organizational role during the conflict. In November 1915, Borel was called by Painlevé to take the head of the new direction of inventions for national defense, and Volterra had in charge numerous inter-allied missions to coordinate the strategies of the countries of the Entente. I refer the reader to the book (Mazliak and Tazzioli, 2009) for getting more details on the war time. As is well known, First World War was an event which touched the whole society of the various countries involved in the conflict and when the guns stopped firing in November 1918, the European scenery presented a desolated picture of ruins, of poor, wounded and deplaced people and, first and firemost, of cemeteries.

[28] Mi permetto, On.le Sig. Prof.re, di augurarLe ottima villeggiatura. Ho scritto da qualche tempo al Sig.Pérès, ma non ho ancora avuto risposta; probabilmente anche lui sarà stato richiamato. (21 Agosto 1914)

Conclusion: a new post-war configuration

In spite of the hard situation at the end of the war, one may think at first glance that there was no real reason that the scientific exchanges would not resume as a prolungation of the pre-war organization. And in fact, it seems that it was attempted to do so. In April 1919 the commissions for the David-Weill and for the Commercy foundations met to decide about the attribution of the grants for the Academic year 1919-1920.

The reports of the meetings show that many difficulties immediately appeared. In the first place, there were endless discussions about the amount of the grants. The disastrous financial situation of the European countries due to the huge debts contracted during the war led to a galloping inflation and comments on the exchange rates are numerous in the correspondence between academics of the time, complaining about the terrible costs of transportation, books, food… The amount of 3000 francs for the David-Weill grant was no more sufficient for a one-year stay anywhere. The Council of Paris University decided that the grants offered by the David-Weil foundation will subsequently be used for one-semester travels.

Another problem was how to deal with the great heterogeneity of the candidates: in addition to the obvious presence of young candidates, there were many of the older ones which had been deprived of the education they deserved because of the conflict. Some students had been severly wounded during the war, as it had been for instance the case for Gaston Julia - see (Audin 2011) - and the commission did not want to commit an injustice by not taking that fact into account. Moreover, the commission had to decide what was to be done about the candidates who had obtained a grant for the year 1914-1915 and had not been in position to benefit from it because of the events.

In the first two years following the end of the war, the commissions tried to face the brain-teaser. An unexpected complication came from the fact that the war had also desorganized the universities and created a shortage of candidates for teaching. Vessiot wrote in his report for the Commercy grant in 1920:

> *The shortage of valuable candidates having a thesis for the vacant or to be vacant positions in mathematics is too worrying to avoid giving all facilities for work even to young mathematicians who just begin to do research*[29].

Therefore, several potential candidates for the grants were nominated as assistants in universities and encouraged to defend their thesis as soon as possible to get a permanent position.

The case of Soula is representative. He had renounced to his project of travelling to Rome and prefered to follow Paul Montel's advice to complete his thesis as quickly as possible. He asked for a Commercy grant for allowing him to live for some time *in Paris* in

[29] Le manque de candidats docteurs, de réelle valeur, pour les postes de mathématique qui sont ou vont être vacants dans les Facultés est trop inquiétant pour qu'il ne soit pas nécessaire de donner, même à de jeunes mathématiciens qui débutent dans la recherche scientifique toutes les facilités de travail possible.

order to finish his doctorate. Borel supported his demand and Soula obtained the grant in 1920, defended his thesis and finally obtained a position at Montpellier in 1921.

Obviously, there were also students who decided to travel. Some of them went to Rome. In 1919, a grant was attributed to Louis Sartre, who had been a prisoner during all the war. He came indeed to Rome, but he gradually turned his back on his mathematical interests and, on his return to France, began a long career in industry. Some of his letters to Volterra explaining the situation have a bittersweet tone, presenting the sojourn as a partial failure. Naturally, we must not overinterpret Sartre's point of view and conclude that the time of happy journeys to Rome was over. Several students sent to Volterra from France found their stay quite successful. One of the post-war students, Szolem Mandelbrojt (1899-1983), who went to Rome from Paris in 1924, had left a vivid memory of his visit to Volterra (Mandelbrojt,1985):

> *After three months, I knew a little Italian language and Volterra asked me to replace him for a lecture because I had already published things about functionals who seemed very amusing to him. Besides in two or three books, he spoke very favourably of the results I obtained in Rome. He treated me as his own son ; he was really extraordinary*[30].

In 1925, when Mandelbrojt had come back to Paris, Volterra wrote to him on 18 March 1925:

> *Everyone remember you here ; the professors and the young scientists and the students. The institution of the small seminar whose creation is due to you successfully goes on and brings good fruits*[31].

Another visitor to Rome and Volterra in the 1920s was André Weil:

> *Arriving in Rome after a month traveling, already able to speak Italian passably, I felt quite at home here, all the more so because the renowned Vito Volterra extended me a fatherly welcome. Though probably less universal than Hadamard, he was an admirable man in all respects. (...) I believed I had some ideas on linear functionals. However shapeless these ideas were, Volterra would lend his ear with tireless patience. (Weil 1991, pp.47-48)*

[30] Après trois mois, je connaissais un peu l'italien, et Volterra m'a demandé de le remplacer dans son cours parce que j'avais déjà publié sur les fonctionnelles des choses qui lui paraissaient très amusantes. D'ailleurs dans deux ou trois livres, il parle très élogieusement des résultats que j'ai obtenus à Rome. II me traitait comme un fils; il était vraiment extraordinaire.

[31] Tout le monde vous rappelle ici [!] ; les professeurs et les jeunes savants et les élèves. L'institution du petit séminaire qui doit à vous sa fondation, continue heureusement et rapporte ses bons fruits.

These testimonies prove that Volterra was still very much interested in supporting young people. However, the 1920s happened to be quite different from the 1910s and the aforementioned failure of Sartre may at least reveal that a part of the initial energy of the tradition evoked by Hadamard in 1916 had faded.

In the first place, Volterra's scientific interests were slightly shifted. Even if he had not completely left functions of lines - Pérès and Volterra published their common book on permutable functions (Peres and Volterra 1924) in 1924 - he was more and more absorbed by other mathematical interests such as mathematical biology - on that topic, consult (Israel 1988) and (Israel and Millan Gasca 2002). In the aforementioned letter to Mandelbrojt, Volterra mentioned that this new work on biology *amused* him a lot. Besides Volterra's way of dealing with functions of line was gradually replaced by the general treatment of functional spaces using methods of general topology and abstract differential calculus. The golden time of the functional analysis *à la Volterra* was now finished. A proof of that fact can be found in the lack of interest for the works of Paul Lévy among French mathematicians in the beginning of the 1920s. Lévy clearly was at the time the best positioned mathematician in France or even in any place to embody the renewal of the discipline, and the magnificent lectures on functional analysis (Levy 1922) he published in 1922[32] made of him the natural heir of Volterra and Hadamard. But the book did not receive the attention Lévy had expected; he reminded of his disappointment in his autobiography (Levy 1970, p.63). It is in fact in a quite different direction, in the theory of probability, that these works would find a new life through the impulsion of Norbert Wiener (1894-1964) when he built the measure of the Brownian motion in (Wiener 1923) and naturally of Lévy himself who became one of the stars of probability theory of the 20th Century - see (Mazliak 2011) and (Barbut, Locker and Mazliak 2013).

A second explanation for the fading out is related to the institutional situation. The financial incapacity of European countries was part of the creation of the International Educational Board financed by the Rockfeller foundation in the United States. The enormous influence of this event on the scientific life between the two world wars has been studied by Siegmund-Schultze in (Siegmund-Schultze, 2001). The great concern of American mathematicians at that time was to stress the relations between mathematics and their application (especially Physics which in the 1920s faced major developments). Before his exploratory trip to Europe in 1923, Wickliffe Rose who presided the newly founded board asked Birkhoff to mention possible privileged contacts in Europe. Birkhoff mentioned Levi-Civita in Rome, and not Volterra[33]. Levi-Civita was younger than Volterra, but this is not the only explanation ; in Paris for instance, Birkhoff indicated Borel as the man to be in contact with. The reason for Birkhoff's choice was probably that Levi-Civita was interested in the most recent developments of mathematical physics such as relativity - see (Nastasi and Tazzioli, 2005). Moreover, he had displayed strong internationalist

[32] The book (Lévy, 1922) is the development of the lectures Lévy made at the Collège de France in the academic year 1919-1920 for the Peccot's foundation.

[33] Letter from Birkhoff to Rose, 19 November 1923. See (Siegmund-Schultze 2001), p.37.

feelings and refused the harsh politics of ostracism towards Germany and its allies dictated by the International Research Council ruled by Emile Picard after the war. As soon as 1922, he accepted to go to Austria for a congress on fluid dynamics in Innsruck - see (Eckert, 2007), p.97. He was the only prominent scientist from the allied powers present there and subsequently became one of the leading figures in the creation of the International Congresses of Applied Mechanics in 1924 - see (Battimelli, 1996). The generous amount of the grants offered by the Rockfeller foundation allowed long research stays exceeding sometimes one and half year. In (Siegmund-Schultze, 2001; pp.288-301), Siegmund-Schultze provided the complete list of the 130 European students who benefited from IEB grants until 1945. Among those sent to Italy, 12 came to Levi-Civita and only 3 to Volterra (Mandelbrojt was the first, followed by Robert Mazet in 1926 and Marcel Brelot in 1929). Obviously, Levi-Civita had become the new main attractor for young mathematicians in Rome. Besides, even if he came officially to Volterra, Mandelbrojt met Levi-Civita in Rome and became close to him[34]. This was also the case for André Weil and Robert Mazet. They were officially sent to Volterra, as it is registered in the lists for the grant attribution. But Ernest Vessiot (1865-1952), the vice-director of the Ecole Normale who succeeded to Borel at this position in 1920, introduced them personally to Levi-Civita.

Last but not least, another argument explains why Volterra had not continued in the 1920s the movement inaugurated 10 years before. The quick evolution of the political situation in Italy and the open opposition of Volterra to Mussolini's regime became soon insuperable obstacles for him to play the role of promoter of cultural and mathematical exchanges between Paris and Rome. The forthcoming paper (Capristo, 2014) is devoted to the relations between Volterra and the Fascist regime and I shall not develop that subject here.

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[34] Mandelbrojt comments on his meeting with Levi-Civita's in (Mandelbrojt 1985) (p.9).

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