Revista Brasileira de História da Matemática - Vol. 9 nº 17 (abril/2009 - setembro/2009) – pág. 7-12
Publicação Oficial da Sociedade Brasileira de História da Matemática
ISSN 1519-955X

REFLECTIONS ON THREE CENTURIES OF LEONHARD EULER

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"In the year 2007, the whole world commemorated three centuries of the birth of the Swiss Leonhard Euler, possibly the greatest mathematician of the 18th century." This is an invented statement, but, on a first inspection, it seems to be a fair description of things that are well-known to the point of being taken for granted: Indeed, Leonhard Euler was born in 1707 in Basel, Switzerland (having lived for almost the entirety of the 18th century), so that 2007 marked his tercentenary; the man and his work were in fact remembered in countless commemorations, all of them being certainly well-deserved, given that he really was a remarkable mathematician. In this way, the opening statement of this note could well pass as nothing more than a description of a fact worth mentioning.

Fortunately, each assertion of that statement may be subject to historical problematization, vesting Leonhard Euler's tercentenary with much more substance than mere celebration – beginning with the idea of commemoration itself. Ephemerides are, sure enough, opportunities to discuss a legacy, perhaps even to criticize it, but they also give way to a reflection on the very nature of history as a discipline and the way in which it incorporates such *commemorative practices*, which are related to (more than anything else) the dimension of memory and to the representation of the past – collective, constructed and interest-laden. As an observer acutely remarked, commemoration seems almost opposite to the historian's task: "In claiming to strengthen solidarities, to negotiate collective identities, to establish the boundaries of groups on the basis of intellectual or institutional legacies, commemorations must remain in implicit tension with analytical and critical history, which always deconstructs alleged solidarities to reveal conflicts and contradiction" (Maier 2000, ix-x). The essential risk is that commemoration thus transforms itself into a teleological feast, celebrating the "precursors" of an invited few, in an effort of memory that excludes their rivals.

An example of the risk that commemorative memory may "kidnap" its object is clear in our opening statement – the "mathematician" Euler. Obviously, it is our right to evaluate the place and the role of Euler in mathematics as a discipline. By all means, however, we cannot forget about the instability of disciplinary and professional boundaries,

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lest we freeze Euler in a static foundation myth. It should be remembered that his first professional placement was in the physiology chair of the Academy of Saint Petersburg – and it is also worth pondering, in his own age's terms, on the very nature of such a placement, that could be easily and anachronistically mistaken for a "professorship". Also, we should not loose sight of his ties to celestial mechanics, astronomy, musical theory, optics, continuum mechanics and even Newtonianism itself (taken as a general philosophical vogue), if we are to gain a deeper understanding of Euler, his work, and his age (for a recent appreciation of Euler's connection with "physics" – a blurry category too – cf. Suisky 2009).

These remarks lead us directly to another problem with our opening statement: What does it mean to speak of Euler as a "Swiss"? We should ask in what degree the idea of "Swiss", with its inevitable flare of modern nationality, makes sense to describe an individual of 1707, or 1750, or even of the late 18th century (the problem of establishing a meaning to the idea of nationality before the 19th century is evidently general, not only Swiss; cf. Hobsbawm 1992). It is undeniable that Euler was born in Basel, which as we know is located in Switzerland, and spent there his formative years (only to leave it forever at 20). But 1707 Basel was located in the Swiss Confederation, an entity that was formally separated from the complex dominions of Holy Roman Empire only after the Thirty Years' War, around the middle of the preceding century, and which was divided in virtually independent cantons that served as unities of political, linguistic and religious identity. In 1712, Catholics and Protestants are still fighting each other with armies (something they will do again as late as 1847), and they are also signing conflicting treaties with France and Prussia. Until complete federalization, in the mid-19th century, it is not unusual that individual cantons refuse to renew the confederate oath, and it is only in 1762 that the first distinctively patriotic association is founded (for a detailed discussion of Swiss nationalism and its founding myths, cf. Zimmer 2003, chapters 1 and 2). To these considerations we should add the necessity of a deeper examination of Euler's ties to Russia and Prussia, places where he spent much more time than in his native Basel. What we should do is aim at a full-fledged history of the complex relationships between intellectual emigrés, such as Euler or Rousseau, and their Swiss "homeland", as well as their posthumous incorporation in the national pantheon.

Another delicate issue, unavoidable though in any approach to Euler's work, is the sensation of irreducible singularity that it engenders – something that points to another assertion present in our guiding statement, about Euler as "the greatest" (mathematician?) of his century. We must nevertheless insist in finding problems: In which sense is Euler to be taken as greatest? In his output? Indeed, Euler's complete works, being edited since 1911, and presently adding up to around 80 volumes, with tens of thousands of pages, cannot be viewed as anything less than hyperbolic. But his age is given to exaggeration, after all: Voltaire's complete works, in the most recent critical edition (Oxford), have reached 142 volumes; just of the *Natural History*, Buffon left us 36 volumes (and hoped to reach fifty); Telemann composed more than 1700 cantatas – and some hundreds of chamber pieces, concerts and symphonies for orchestra, operas, dances, oratories and masses. The list of contemporary excesses is itself infinite. Euler's singularity could be located, then, on the variety of themes, not on sheer size – but, again, the hypothesis does not stand

comparison with multiple examples of his own age. Perhaps, then, we should take his unmatched influence on future generations? Well, but what to say then about the Bernoullis, d'Alembert (himself with imposing 40 volumes of complete works!) or Lagrange?

We should ponder, finally, on that which might be taken as the most certain (because of its "objectiveness") assertion of the initial statement that has been guiding these considerations – the assertion of Euler as belonging to the 18th century. Naturally, from a strictly chronological point of view, Euler's life and work are clearly bounded in the period that goes from 1701 to 1800. But, as we know much too well, a century as an analytical, not just time-reckoning, unit, is very different from the uniform duration of 100 years. The English "long 18th century" extends from the 1688 Revolution to Napoleon's defeat, at least, reaching, according to some, the political reforms of the 1830s (O'Gorman 1997); in France, the "century" begins much later, with Louis XV's ascension, and meets its end in 1789. This fluidity of temporal boundaries, and more than that, the fluidity of historical and cultural contents that are elected as determinatives (this being the root of any periodization) spreads to that idea which is the sibling of the idea of 18th century, at least in European contexts: the Enlightenment. Euler is frequently seen against the background of a coherent movement, a general Enlightenment of which he is a would-be foe, mainly because of his proverbial religious fervor. The problem is that this Enlightenment that finds a pillar in irreligion is a phenomenon far from hegemonic. We are reminded by Yvon Belaval that there are many Enlightenments, not only that one mainly identified with French "Lumières": there is the English "Enlightenment", the German "Aufklärung", the Russian "Prosveshenie". Each of them has its chronological marks; each has its obsessions: Religious tolerance, despotism, the critique of knowledge, materialism (Belaval 1978). Euler moves, in Saint Petersburg and Berlin, in spaces in which his devotion is simply not a "problem", as it might be in Paris. Peter Gay seems to be the one who better grasped the false opposition between Euler and this unqualified Enlightenment: "For every infidel mathematician there was a pious one, for every d'Alembert there was an Euler" (Gay 1995, p. 338).

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Motivated by such questions, we organized, together with Sergio Nobre (UNESP) and Cláudio Possani (USP)[1], the Leonhard Euler (1707-1783) Tercentenary Brazilian Meeting. The event took place in December 5, 2007, in the School of Arts, Sciences and Humanities (EACH) of the University of São Paulo, and was an initiative of the Brazilian Society for the History of Mathematics (SBHMat), with support from the University of São Paulo (through EACH and the Institute of Mathematics and Statistics) and from the International Commission on the History of Mathematics (ICHM), and also with the gracious collaboration of Olimpíada Paulista de Matemática and Colégio Leonardo da Vinci (Jundiaí, SP). The Meeting counted with guest speakers as well as submitted oral and

[1] This was the Organizing Committee. The Scientific Committee included Carlos H. B. Gonçalves (USP), Clóvis Pereira da Silva (UFPR), Francisco César Polcino Milies (USP), John Fossa (UFRN), Sérgio Nobre (UNESP), Sueli Costa (UNICAMP) and Suely Druck (UFF).

poster presentations, the latter mostly by graduate and even undergraduate students involved in research on history of mathematics. We take this as a very positive sign of the vitality of the field.

In 2008 we received from SBHMat the generous invitation to edit the present issue of *Revista Brasileira de História da Matemática (RBHM)*, with the works presented in the guest lectures and oral presentations that took place in the Meeting[2]. Not all of the speakers were able to contribute with the written record of their lectures, but this issue nevertheless remains as an excellent sample of the rich historiographical and mathematical discussions that animated the event.

In his paper, which is to be found right after this brief introduction, Ubiratan D'Ambrosio sketches a biographical profile of Euler, accompanied by a rich, commented Eulerian iconography (composed by portraits and frontispieces of his works). Besides the attention to iconography, rightly recognized by D'Ambrosio as one of the most important forms of contemporary witness to Euler, the paper also gives the reader the possibility of knowing some of the written sources that are essential to an understanding of the image of Euler that the 18th century was already forming, like the texts of Nikolaus Fuss and Condorcet. As the author remarks, this image is forged in parallel to that of Enlightened Russia. The paper also brings a reflection on Euler's influence on methodological and pedagogical traditions, and even in the author's own path.

Afterwards, Circe Mary Silva da Silva presents the results of an original work of reconstruction of the conditions that resulted in the publication and circulation, in Joanine Brazil, of a translation into Portuguese of Euler's well-known algebra textbook (a presentation that is preceded by a general description of the book's contents, and followed by a detailed analysis of the way in which Euler treats complex numbers). The author reports her fascinating quest, first for the translated book, exceedingly rare, and then for its elusive translator, who is not named in the volume. We are taken to a world of newspapers, official decrees and contemporary reports on the Military Academy of Rio de Janeiro (according to the book's frontispiece, this institution would have officially adopted it in its curriculum), and also to a universe of mathematics textbooks competing for didactic space in the first decades of the 19th century, an universe which will eventually exclude Euler's book in favor of Lacroix's. Following the author, we gain a deeper understanding of the world of the press in the Court, mathematics education in the Academy, governmental interferences, the conditions of access to specialized knowledge and the vehicles chosen to do that, and, above all, the trajectory of Manuel Ferreira de Araújo Guimarães, a figure who, in a remarkable work of historical investigation, Silva convincingly presents as the unknown translator of Euler's algebra into Portuguese.

In his article, Rogério Monteiro de Siqueira examines the interpretations given by the 20th century of one of Euler's most famous results, namely the formula connecting the number of faces, vertices and edges of a polyhedron (presented in two works published in

[2] Only the last two papers published here were not presented in the Meeting, and were independently submitted to *RBHM*. Being translations with commentaries (and preceded by introductions) of works by Leonhard Euler that are unavailable in Portuguese, evidently related to the rest of the issue by their content, the editorial body understood that it would be in the interest of readers to publish them here as well. We should stress that all the papers went through the normal refereeing process of *RBHM*.

1758). Rather than interpretations, maybe one should say that the article deals with the attempts of appropriation of Euler's formula – and even his historical figure – by two important traditions that strive to lay their foundations in this formula: combinatorial topology and discrete geometry. Beginning with a close reading of the original works, as well as the letters on the subject sent by Euler to Goldbach, the author offers us a detailed picture of Euler's declared intentions when dealing with the problem of determining the "general properties of solids limited by plane faces" and the methods he employed to pursue such intentions. From this, Siqueira analyzes recent attempts, carried on by researchers associated with each of those traditions, at claiming a higher faithfulness to Euler's original purposes and at highlighting, in his 18th-century interests and methods, foundational forebodings of their own fields. Even though this occurs ostensibly in a mathematical level, with different generalized notions of polyhedra being presented by each area as more faithful to Euler's conception, the author skillfully suggests to us that there is also a definite agency of non-epistemic, but sociological, values (as far as this separation is possible) in the success with which topologists present themselves as the "legitimate heirs", excluding mathematically acceptable alternatives for the definition of the object, as those proposed by certain researchers in discrete geometry.

It is exactly this old relationship between topology and Euler's formula for polyhedra that is dealt with by Daciberg Lima Gonçalves in his paper. Without intending to find in Euler a topological spirit, Gonçalves presents us an informative conceptual history, free from such anachronism – rather, he is interested in showing, from an internal point of view, how the formula for polyhedra was gradually generalized into a complete tool for the classification of surfaces, resulting in a topological invariant that can be shown to be the only of its kind (formed from the numbers of "vertices", "faces" and "edges" of a closed surface, or yet, from one of its triangulations): the so-called Euler characteristic. Advanced results in topology from the 20th century (and even from the present decade) are presented in a clear way, arriving at the generalization of the Euler characteristic to other topological spaces, including manifolds, through homological considerations and the idea of characteristic classes.

The last paper presented in the Meeting and published here is by Nelo D. Allan. In this paper, the author clarifies, in all detail, the contents of three long and difficult works by Euler on the so-called pairs of amicable (or friendly) numbers, pairs in which each element equals the sum of the proper divisors of the other. Frequently considered as a chapter of recreational mathematics – itself unfairly subject to value judgments by some who consider it as secondary in realtion to "deep results" –, the investigation of amicable pairs is a legitimate part of Euler's incursions into number theory, as Allan clearly shows. In the three works that Euler published on the subject, he applied himself to the search of proofs and general properties with the same willpower that has sometimes been seen as defining an Eulerian style.

Finally, this issue of *RBHM* closes with two papers that introduce, translate directly from Latin, and comment on two works of Euler. The first paper, signed by John Fossa and Sarah M. S. Leôncio, deals with a publication from 1747, in the *Nova acta eruditorum*, entitled "De numeris amicabilibus" (On Friendly Numbers) – one of the sources of the preceding study by Nelo D. Allan. The other paper, authored by us, deals

with Euler's "Theorematum quorundam ad numeros primos spectantium demonstratio" (Proof of Certain Theorems Concerning Prime Numbers), published in 1741, in the *Commentarii academiae scientiarum Petropolitanae*, containing a proof of the so-called Fermat's little theorem, and which is also part of the Eulerian production in the number theory.

We wish everyone a good reading.

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