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CAMILLE FLAMMARION'S PYTHAGOREAN MOMENT

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Abstract

Flammarion's Woodcut is a frequently discussed engraving in Camille Flammarion's 1888 work on meteorology. It is interpreted herein as a symbolic presentation of the (Pythagorean) thesis that mathematics is the key to knowledge and, specifically, that the Calculus is the key to meteorological knowledge.

Keywords: Mathematics, History, Flammarion Woodcut, Pythagorean type of Philosophy.

[MOMENTO PITAGÓRICO DE CAMILLE FLAMMARION]

Resumo

O Woodcut de Flammarion é uma conhecida gravura do trabalho, de 1888, de Camille Flammarion sobre a meteorologia. É interpretada aqui como uma apresentação simbólica da tese (pitagórica) de que a matemática é a chave ao conhecimento, e, especificamente, de que o cálculo é a chave ao conhecimento meteorológico.

Palavras-chave: Matemática, História, Woodcut de Flammarion, filosofia pitagórica.

Introduction

While science (in the root sense of this word) can at least make truth claims, *mythopoesis*, on Plato's view, and by extension all art, only tells a convenient and likely story. We do not take this to be a disparagement of art by Plato, the *philomythos*, but rather an exploration of another dimension of man's humanity and an expression of his existentialist view of man:

existence precedes essence (as the Sartrean mantra would have it), that is, men are not some quiddity determined by a fixed nature, but historical beings that that acquire their natures through their lived experiences.

This being the case, we take Plato at his word and offer our probable and convenient account of the "Flammarion Woodcut" (see Figure 2), an engraving[1] much discussed and oft reproduced in the literature. Hopefully our account will be likely in that it removes the paradoxicality from the standard accounts. It will also be convenient in that it will capture Flammarion's at least momentary conception of science in a distinctly Pythagorean mold.

Camille Flammarion

Before turning to the Woodcut, it seems appropriate to say a few words about Flammarion himself. (Nicolas) Camille Flammarion came into the world on the 26th of February, 1842, in the village of Montigny-le-Roi (Haute-Marne) in the eastern part of France. The firstborn of four siblings, Flammarion was a precocious child, dominating both reading and arithmetic at a young age. He studied with the Jesuits, at first in the provinces and later in Paris. By the time he was fifteen, he was working as an apprentice engraver, which is significant for our purposes since the Woodcut in which we are interested, although not actually executed by him, was of his design.

Flammarion himself, however, stated, to a reporter from *McClure's Magazine* (SHERARD, 1894), that his greatest interest had always been astronomy. Miller (1925) reveals further important details. In 1847, when Flammarion was only five years old, his mother (who wanted him to become a priest) arranged for him to view an eclipse of the sun reflected in a pail of water. He was also able to view another eclipse in 1851, this time through a shard of smoked glass. Upon learning that eclipses could be predicted using mathematical calculations, he resolved to master the mathematics necessary to do so and, undoubtedly, internalized the view that mathematics was the key to scientific knowledge.

When only sixteen years old, he entered the Paris Observatory, where he remained for four years, making observations of comets. On leaving the Observatory, he came to the attention of the public as an eminently literary popularizer and public lecturer on astronomy. Eventually, he was able to equip himself with a small telescope so that he could make observations from his own apartment, but later returned to the Paris Observatory, where he made notable progress on the observation of double and multiple stars.

[1] Whether the image be a woodcut or an engraving is quite immaterial to our argument and, thus, we will use the two terms interchangeably.

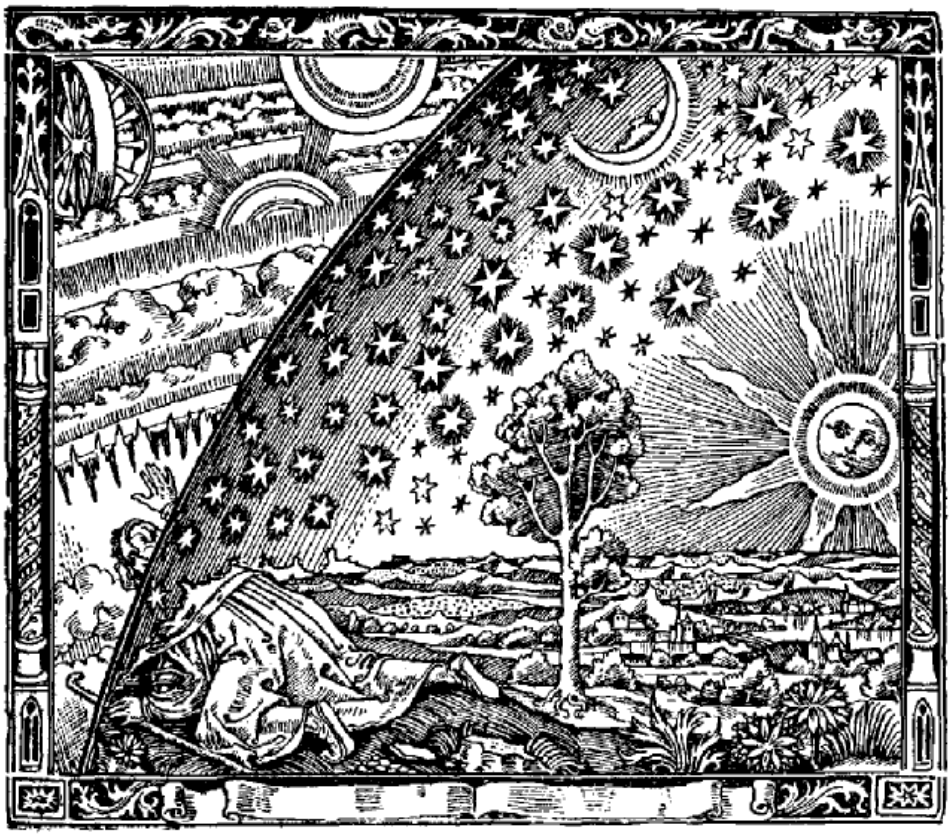
Figure 1. "Flammarion at his Telescope."
Font: (SHERARD, 1894, p. 577)

Flammarion also became interested in meteorology and even made various meteorological observations in hot air balloons. In fact, the book, in which the Woodcut appears (FLAMMARION, 1888), is a popularization of meteorology. In it, he expressly affirms that meteorology had become, in his age, a true and independent science.

In 1883, Flammarion founded the observatory at Juvisy-sur-Orge in the environs of Paris (see Figure 1). He also added physics and Spiritism to his role of writings. In fact, he was a friend of Allan Kardec (Hippolyte Léon Denizard Rivail, 1804–1869), the codifier of Spiritism, rendering a long funeral oration to him at his burial. He founded, in 1887, the *Société Astronomique de France*, which continues to this day to be an important scientific organization. Just thirteen years before his passing away at 83 years of age, he received the prestigious *Ordre National de la Légion d'Honneur* for his work in astronomy and the popularization of this science.

The Woodcut at First Glance

Like some old black and white movies, the Woodcut has been re-released in color and/or remade with changes in details (the "missionary", for example, is sometimes portrayed as poking his head out of the right side of the celestial sphere, rather than the left side). Although there is some confusion about the origin of the Woodcut, the original seems to be the version printed in Camille Flammarion's 1888 popularization of meteorology. The confusion regarding the provenience of this graphic image may be due to Flammarion's caption, which reads: "A missionary from the Middle Ages recounts how he came upon the point where the sky and the Earth come together …" (See Figure 2.).



Un missionnaire du moyen âge raconte qu'il avait trouvé le point où le ciel et la Terre se touchent...

Figure 2. The Flammarion Woodcut.
Font: FLAMMARION, 1888, p. 163

In any case, the image is supposed to represent a medieval cosmology, in which the Earth is flat and the universe bounded by a spherical firmament. How could it be that, in this view, the sky and the Earth come to touch each other? We could suppose that the flat Earth just stretches out until it abuts the sky. Such an unaesthetic supposition, however, hardly qualifies as medieval.[2] A likely model would be that of a cube inscribed in a sphere, as shown in Figure 3, a detail from a drawing by the German artist Augustin Herschvogel (1503–1553), taken from the MathIsFun (2014) page. The top of the cube would be the inhabited Earth. Thus, there would be four possible points where the inhabited Earth and sky meet: the four points of internal tangency where the top face of the cube touches the sphere (presumably we would not have access to the bottom face of the cube).

There is a slight inconvenience in this model, because, in it, the inhabited part of the Earth would not be at the exact center of the universe, which would coincide with the center of the sphere. To remedy this inconvenience, we may suppose that the top half of the inscribed cube has been cut away, so that the inhabited part of the Earth is the top face of the resulting half-cube.

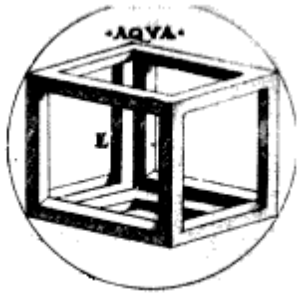


Figure 3. A cube inscribed in a sphere.
Font: MathIsFun (2014)

We consider the half-cube model first. Figure 4(i) shows a cross-section of the universe at the equator of the sphere, so that we would be looking down upon the Earth (the square) from above. The center of the inhabited part of the Earth, presumably Rome, would then coincide with the center of the sphere and the segment $\overline{RA}$ would be the radius of the celestial sphere. As the figure shows, however, the top face of the half-cube does not touch the celestial sphere. It is true that the bottom face of the half-cube touches the sphere (in four distinct places), but that means that the missionary would have to had burrowed down through the Earth to get at the bottom face, which does not seem to be consistent with the

[2] The view that the Earth and the sky come together was a medieval conception, albeit a rather naïve and pedestrian one, which Flammarion cites in other instances. The model of the universe upon which it would have to be based by erudite standards would have to be aesthetic, mathematical and undoubtedly of Greek or Roman provenance.

graphic image of Flammarion's Woodcut.[3] Thus, we are led to reject the present model and adopt the full-cube model.

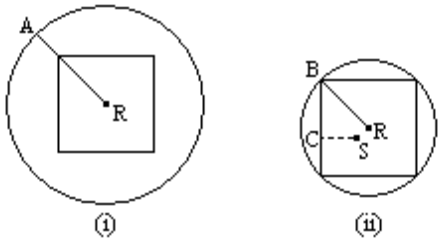


Figure 4. Cross-sections of universe: (i) half-cube model and (ii) full-cube model.

The cross-section of the universe containing the top face of the full cube is given in Figure 4(ii). Rome (R) is still at the center of the inhabited Earth, but is in a slightly eccentric position in relation to the center of the celestial sphere. In fact, it is directly above the center of the sphere, at a distance of half the height of the cube. The radius of this cross-section is $\overline{RB}$, which is smaller than the radius of the sphere since the present cross-section is not at the sphere's equator, but some way above it. Since B is a point of tangency of the cube to the sphere, the missionary could poke his head out into nothingness at this point.

We make here two asides. The first is that, although Rome is no longer at the center of the universe in this model, this is somewhat compensated for by the fact that the center of the Earth (that is, the center of the cube) does coincide with the center of the sphere. Secondly, when Columbus set sail from Spain (S in Figure 4(ii)), traversing a westerly route across the Atlantic (the broken segment $\overline{SC}$), he should have been expecting, according to this model, to fall off the Earth at point C. Of course, he did not have a flat Earth model, but, apparently, many of his prospective shipmates did!

In the Cosmic Attic

Returning to the Flammarion Woodcut (Figure 2), we see at once that the intrepid missionary did not actually poke his head out into nothingness – indeed, how could he! At first blush, the wheels and spoke-like imagery lead us to think that he got at least a glimpse of the cosmic gearbox: Plato's spindle of necessity, etc. A closer look, however, reveals no spindle, no circle of the Same, nor one of the Different. Where are the Sirens and how do the spinning planets keep spinning? Indeed, the whole scene is curiously devoid of motive power. Does this static quality indicate that the cosmic gearbox has broken down? Do then the sun, moon and stars, portrayed as duly performing their daily tasks, continue revolving

[3] There is the logical possibility that the missionary found, say, an interconnecting series of caves that led him to the bottom face of the cube. Since there is nothing in the Woodcut that indicates this possibility and since it would not materially affect our subsequent analysis, we will not entertain this hypothesis here.

out of habit? Or have we here a parody on ancient cosmologies in the face of the Newtonian theory of universal gravitation?

If it's a parody, perhaps we are faced with an elaborate cartoon. The missionary has stumbled into the universe's attic, the cosmic junk box wherein spare parts are stored and worn out pieces of cosmic equipment are tossed out and forgotten about.

The foregoing interpretations trade off the cosmic for the comic. It is, however, a short-lived comedy that merely elicits a wan smile and does not capture the imagination. They are not likely and convenient tales that can explain why the Woodcut fosters so much interest and debate amongst many philosophically diverse people. A more interesting possibility is that suggested by Pavel Gregoric (2010) who points to Simplicius' third-hand (by way of Eudemus) account of an argument originally propounded by Archytas (428–347 a.C.). If the universe were finite, so goes the argument, it would have a boundary or limit (the celestial sphere) and anyone arriving there could just reach out with his staff or his hand and, thus, get beyond the finite universe. Since this is impossible, the universe must be infinite. Indeed, the missionary in the Flammarion Woodcut is pictured as reaching out with both his staff and his hand (as well as, for good measure, his head) and, thus, the Woodcut, according to Gregoric is a graphic image of Archytas' paradox.

Although *prima facie* plausible, the paradox can be met in various ways. We mention but two. First, from a commonsense point of view, it could be argued that, were we to get to the boundary, our natural expectation of being able to reach out across it would be frustrated by the very nature of the boundary as limit: one just could not crack through the cosmic sphere. Second, from the point of view of contemporary physics, the universe is finite, but unbounded, so one would never be able to arrive at any supposed boundary. We must admit, nevertheless, the fragility of the Archytacian argument is not a very strong argument against Gregoric's theory: it still may have been Flammarion's intention to make a graphic image picturing the paradox.

A Symbolic Interpretation

All the foregoing, however, is but propaedeutic, a setting of the scene, as it were, for our likely story. In it, we see a missionary whose head and hand and staff are thrust into a static other world, and whose torso is simultaneously whirling about on the celestial whirligig. How are we to suppose that he has survived to tell the tale?

Any way that we try to make sense of the Woodcut as picturing a happenstance in physical reality is fraught with senseless consequences. We propose, therefore, that, in contrast to Columbus' prospective shipmates who were about to fall *off* the Earth, the missionary fell *out* of the Earth into another world of static, unchanging relations. Flammarion (1888, p. 162) confirms this interpretation:

> "(…) *Un naïf missionbaire du moyen âge raconte même que, dans un de ses voyages à la recherche du Paradis terreste, il atteignit l'horizon où le ciel et la Terre se touchent, et qu'il trouva un certain point où ils n'étaient*

> *pas soudés, où il passa en pliant les épaules sous le couvercle des cieux… Or cette belle voûte n'existe pas!"* (FLAMMARION, 1888, p. 162).

If so, the symbolic values of the missionary's head, hand and staff are fairly clear. The head is the seat of understanding and thus symbolizes man's theoretical understanding of the universe. Both the hand and the staff are instruments of power, especially the power of man to dominate the world. Indeed, the hand is used to dominate by force (wrest, punch, slap, *etc*.), but also to fiddle with things, taking them apart and putting them back together again. Similarly, the staff can be used as a weapon or as an instrument to help man in his meanderings of discovery. Thus, they symbolize practice. If this be so, the crouching missionary, poking head, hand and staff into the Otherworld, is the reciprocal dialogue between theory and practice[4] which not only gives him comprehension of the world in which he finds himself, but also gives him power over this world and, consequently, promotes human advancement.

The *locus classicus* of this vision of things is that of Plato's mathematical theory. In fact, for all the ancient Greeks, theory was originally the viewing of the oracle's instruments. On special feast days, the privileged few would make the long and hazardous pilgrimage to Delphi for the viewing that was supposed to impart wisdom to them. So too, the missionary has traveled afar in order to betake himself to a (metaphorical) place from which he can contemplate mathematical truth. Interestingly, the ancient Greek theorist was not expected just to bask in his newfound wisdom, but was enjoined to return to his commonwealth and put his knowledge to work for the betterment of the state. So too, the missionary does not merely poke his head[5] into the other world merely to contemplate it, but also thrusts his hand and his staff into this world also. The hand is man, the builder and transformer, and the staff symbolizes his instruments. Thus, mathematical theory is to be put to use in the physical world.

Actually, we have gotten ahead of ourselves a bit, for we have yet to show, except for a loose analogy to Plato, that Flammarion's Otherworld is indeed mathematical. To do so, we must interpret the elements in the Woodcut that lay beyond the sphere of the universe.

Mathematics and the Otherworld

Interestingly enough, many interpreters of Flammarion's Woodcut claim that it is a vision of man's quest for knowledge, embedded in either the context of the Scientific Revolution or mysticism (more properly, Spiritism). There does not seem to be, however, any detailed account of the symbolism of the otherworldly elements beyond the sphere of the universe. At most, the two wheels in the upper left-hand corner are affirmed to be the double wheels described in Ezekiel's vision at Ezekiel 1.15–21. On the one hand, Ezekiel's vision would be appropriate, since it is generally taken to be an affirmation of the harmony and divine

[4] For more on this, as well as the information to be presented in the next paragraph, see FOSSA (2008).

[5] There may also be latent sexual symbolism here symbolizing man's creative powers.

providence of the universe, thereby conjoining two of Flammarion's major interests, science and Spiritism. On the other hand, the wheels do not really correspond to those in Ezekiel's vision, which have, for example, eyes along their rims. It is not necessary to reject this interpretation, for artistic renditions may have a multiplicity of different meanings at different levels. Rather, we would suggest that there is still needed an interpretation that fits the wheels into a more systematic interpretation of all the various elements in the otherworldly part of the Woodcut. We will now attempt to do this.

One of the first things that struck the present writer about the aforementioned part of the Woodcut is the abundance of apparently meaningless lines that just seem to go on and on without end. The same thing happens with the clouds and what appears to be a line of flames. The underlying motif of endless progression[6] is patent. This motif can be interpreted as an artistic rendition of infinite series. Now, infinite series are intimately related, perhaps even more so in the minds of nineteenth century mathematicians than those of the present day, to the Calculus. Thus, the main artistic motif of the otherworldly part of Flammarion's Woodcut indicates that it is the Calculus that gives man understanding of the universe in which he finds himself. We have, therefore, a modernized form of Pythagoreanism: mathematics – in the form of the Calculus, rather than arithmetic – is the key to the understanding of the universe.

Once this is seen, the wheel within a wheel is easily interpreted as the decomposition of, for example, velocity in mutually perpendicular directions. The technique of decomposing vector quantities into orthogonal components is used intensively in the mathematical analysis of the physical world and is, thus, appropriately utilized by Flammarion to emphasize his Pythagorean message.

The two conjoined circles to the right of the wheels would seem to be two bodies bound by opposite electric charges. In fact, electromagnetic phenomena were being intensively studied in Flammarion's lifetime and, although a greater appreciation of the atom as composed of electrons and protons would have to wait for some years posterior to the publication of Flammarion's book, Coulomb's inverse square law had already been known a hundred years previously. Alternatively, the two conjoined circles could be seen as two bodies held together by the force of gravity. The ambiguity is interesting because gravity is also described by an inverse square law, analogous to that for electromagnetic forces and this reinforces Flammarion's vision of the universe as a harmonious whole, whose basic harmony, albeit, is only revealed by mathematics.

Finally, we should mention the incongruous appearance of clouds in the Otherworld. Although clouds are an integral part of the weather phenomena discussed in Flammarion's book, they should only appear under the dome of the sky and not in the otherworldly part of the Woodcut. Indeed, what is "cloudy" is not clearly seen and, thus, represents ignorance, the exact opposite of mathematics. Perhaps, the other elements of the design are to be seen as hedging in the clouds, or dispelling them from the sight of the missionary. If so, it would mean that mathematics dispels ignorance, leading man to true knowledge. At first sight, this would seem to be a bit cheap, or "kitschy", and, from an

[6] This progression may also represent Spiritism's Law of Progress.

artistic point of view, would not justify introducing clouds into the Otherworld. Upon further reflection, however, it adds a brilliant self-referential critique to the basic design of the Woodcut, since part of the ignorance that mathematics presumably dispels is the very view of the universe depicted therein: that the missionary actually found the spot where the sky and the earth meet! Further, by implication, mathematics dispels the ignorance of pseudoscience and reveals the true structure of the universe.

Conclusion

The interpretation of the Flammarion Woodcut given above is that it is a symbolical presentation of the thesis that mathematics is the key to knowledge about the world in which we find ourselves. This thesis is generally referred to (following Bertrand Russell) as being a "Pythagorean" type of philosophy. In this sense, it is immaterial that the Pythagorean emphasis is on arithmetic, whereas Flammarion's emphasis is on the Calculus. The important point is that mathematics is looked to as providing understanding. It is in this sense that we claim that the Woodcut is a "Pythagorean moment" in Flammarion's book on meteorology (FLAMMARION, 1888). Indeed, the aforementioned book is a popularization of meteorology and, as such, contains but little mathematics. The Woodcut, thus, admonishes the discerning reader that he is only at the threshold of real understanding and that further progress requires a more mathematical presentation.

Pursuant, however, to recent developments in scholarship on the Pythagoreans and related groups, it behooves us to say a few more words regarding what is to be understood by attributing the adjective "Pythagorean" to Flammarion. According to Fossa and Erickson (2014), the Pythagoreans proceeded by using "mathematical allegory", that is, they first elaborated a mathematical structure which they thought interesting in itself and then used (rather arbitrary) heuristic principles to relate the structure to some aspect of reality in a compelling mythopoetic way. For the Pythagoreans, therefore, truth is obtained from the mathematics.

Flammarion, in contrast, as a representative of modern science, elaborates mathematical models of physical situations. This means that he starts from a given physical situation, aspects of which can be expressed by relations between measured physical quantities. These relations are generally expressed by equations and manipulated mathematically. Here mathematics is not seen as the origin of scientific truth, but as truth-preserving reasoning, so that the origin of truth is given by empirical observation. Nevertheless, mathematics is a fundamental instrument that gives science deep insights, which would be unattainable without it.

In labeling Flammarion a Pythagorean, we do not wish to attribute to him procedures based on mathematical allegory, but only the more general attitude of holding mathematics to be central to the elaboration of our knowledge of the word in which we find ourselves. Specifically, we interpret the Woodcut as a symbolic expression of this general "Pythagorean" outlook that meteorological phenomena are to be understood using the Calculus.

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